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SIMPLE PERIODIC RINGS

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Let A be a power-associative ring, and suppose that for each $a \in A$ there exists an integer $n = n(a) > 1$ such that $a^n = a$. Such a ring A is called a *periodic ring*. In this paper the structure of all simple periodic rings of characteristic not 2 or 3 is determined. This solves a problem posed by Osborn [Varieties of Algebras, Advances in Mathematics, to appear]. It follows from these results and from Osborn's that every flexible periodic ring with no elements of additive order 2 or 3 is a Jordan ring.

Let A denote a simple periodic ring. It is shown in [1] that every element of a periodic ring has finite additive order, and it is known that any simple ring has a well-defined characteristic. Thus A must be an algebra over Z_p, the integers modulo p, for some prime p. We suppose that $p \neq 2$ or 3 in order to use the results of [1]. Definitions not given here may be found in [1].

Let the multiplication give in A be denoted by juxtaposition and let the operation "o" in A be defined by $aob = 1/2(ab + ba)$ for $a, b \in A$. The algebra formed by taking the elements of A under the same operation of addition but under the new multiplication "o" is denoted by A^+. It is shown in [1] that A^+ is a simple Jordan algebra and that if A is not a field than A^+ is a periodic Jordan algebra of capacity 2. By a Jordan algebra J of capacity 2 we mean a simple Jordan algebra in which there exist two orthogonal idempotents e_1, e_2 adding to the unity quantity and having the property that the Peirce subspaces $J_1(e_1)$ add $J_1(e_2)$ are Jordan division algebras. Periodic Jordan algebras of capacity 2 are characterized in [1] by

PROPOSITION. *Let Φ be a periodic field of characteristic not 2, let μ be a nonsquare in Φ and let Φ_2 denote the ring of 2×2 matrices over Φ. Then the Jordan subalgebra of Φ^+ consisting of the set*

$$J = \left\{ \begin{pmatrix} \alpha & \beta\mu \\ \beta & \gamma \end{pmatrix} \middle| \, \alpha, \beta, \gamma \in \Phi \right\}$$

is a simple periodic Jordan algebra of capacity 2 over Φ. Conversely, every simple periodic Jordan ring of capacity 2 and characteristic not 2 is isomorphic to J for some choice of Φ.

In view of this proposition we may identify the elements of A with 2×2 matrices of the form