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DENDRITES, DIMENSION, AND THE INVERSE ARC FUNCTION

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In this paper, the concept of an inverse arc function is introduced. An inverse arc function f is a function such that for each arc L in the range of f, there exists an arc L_1 in the domain of f such that $f(L_1) = L$. It is proved that a dendrite D is the continuous image of an inverse arc function f with domain an arc L if and only if D has only a finite number of endpoints. Other results are obtained telling what dendrites can be ranges of continuous inverse arc functions having dendrites as domains.

The dimension raising ability of a continuous inverse arc function whose domain is a dendrite is questioned. It is proved that if D is a dendrite with only a countable number of endpoints, then there does not exist a continuous inverse arc function f with domain D such that $\dim f(D) \geqq 2$. If a dendrite D has uncountably many endpoints, then the question is left unanswered.

Basic theorems and definitions used are as stated in [3], [4], [5], and [6]. In particular, a continuum M is a dendrite provided it is locally connected and contains no simple closed curve. A continuum is a compact closed connected set. Other characterizations of a dendrite are also used. Topological spaces considered are all separable metric spaces. If x and y are distinct points, then xy will denote an arc with end points x and y.

DEFINITION. Let $f\colon X \to Y$ be a function from X onto Y. Then, f is an inverse arc function if and only if for each arc $L \subset Y$ there exists an arc $L_1 \subset X$ such that $f(L_1) = L$.

In this paper the class $\mathscr{D}$ of all dendrites is partitioned into two subclasses, $\mathscr{H}$ and $\mathscr{K}$, such that

$$\mathscr{H} = \{X\colon X \in \mathscr{D} \text{ and there exists a continuous inverse arc function, } f, \text{ with domain an arc } A \text{ and } f(A) = X\}$$

and

$$\mathscr{K} = \mathscr{D} - \mathscr{H} .$$

Then, it is shown that

$$\mathscr{H} = \{X\colon X \in \mathscr{D} \text{ and } X \text{ has only a finite number of endpoints}\} .$$

and