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CYCLIC VECTORS FOR REPRESENTATIONS ASSOCIATED WITH POSITIVE DEFINITE MEASURES: NONSEPARABLE GROUPS

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Let μ be any positive definite measure on a locally compact group, and let $(\pi^\mu, \mathscr{H}^\mu)$ be the associated unitary representation of G. Previous work of the authors' showed that a cyclic vector exists for π^μ if G is second countable; there is now a simple proof of this result, due to Hulanicki. Rather elementary conditions on the way μ is related to the geometry of G are examined which are necessary, or sufficient, for the existence of a cyclic vector. These conditions require μ to be "constant" on cosets (or double cosets) of certain subgroups of G. A conjectured necessary and sufficient conditions is presented. These results are adequate to decide whether or not π^u is cyclic for various nontrivial measures. As a special case it is shown that the left regular representation of G is cyclic $\Leftrightarrow$ G is first countable.

1. Notations. All groups are locally compact, not necessarily second or first countable. The space $C_c(G)$ of continuous functions with compact support is given the usual inductive limit topology. Convolutions $f*g$ of functions in $C_c(G)$ are defined in the usual way; we use the involution operation

$$f^*(x) = \overline{f(x^{-1})}\Delta(x^{-1})$$

(Δ the modular function) which makes $C_c(G)$ a $\|\cdot\|_1$-dense *-subalgebra of the convolution algebra $L^1(G)$. Positive definite measures μ are Radon measures (*not necessarily bounded*), so that $\mu \in C_c(G)^*$, that satisfy the condition

$$\langle \mu, f^* * f \rangle = \int_G (f^* * f)(x) d\mu(x) \geqq 0, \text{ all } f \in C_c(G) .$$

Positive definiteness is indicated by writing $\mu \succ 0$. The representation $(\pi^\mu, \mathscr{H}^\mu)$ associated with μ is defined by imposing the conjugate bilinear form

$$(f, g)_\mu = \int g^* * f d\mu \text{ for } f, g \in C_c(G)$$

on $C_c(G)$. Left translation $\lambda_x f(y) = f(x^{-1}y)$ preserves this form.

If we write $\|f\|_\mu = (f, f)_\mu^{1/2}$, and set $\mathscr{N}^\mu = \{f \in C_c(G): \|f\|_\mu = 0\}$, then the quotient map $j_\mu: C_c(G) \to \mathscr{H}_0^\mu = C_c/\mathscr{N}^\mu$ maps $C_c(G)$ into a