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WILD ARCS IN THREE-SPACE

1: FAMILIES OF FOX-ARTIN ARCS

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Roughly speaking, a Fox-Artin arc is an arc which is tame modulo one endpoint at which it has penetration index three, and which may be constructed in the way that the examples of R. H. Fox and E. Artin were constructed in their classical paper of 1948.

For each oriented Fox-Artin arc, there is an associated infinite sequence of oriented prime 2-component links, which is an invariant of the local embedding type of the arc in R^3. Using existence results from link theory, this result yields the corollary: If M is a 3-manifold and p a point in the interior of M, then there exists an uncountable family of locally non-invertible Fox-Artin arcs in M, which are wild at p.

Later papers will be concerned with developing invariants of the oriented local embedding type of an arc k_n which is tame modulo one endpoint, at which it has penetration index $2n+1$.

0. **Introduction.** The results of this paper originated in attempts to answer the following two questions:

1. Do there exist uncountably many arcs in Euclidean 3-space R^3, which are tame modulo one endpoint? ([1], p. 33. Such arcs are called "nearly polyhedral" in [1].)

2. Are the arcs 1.1, 1.1*, 1.3 of [3] amphicheiral or invertible? (Problem 17 of [2].)

In 1961, R. H. Fox and O. G. Harrold [4] succeeded in completely classifying the Wilder arcs of penetration index 2, and the existence of uncountably many non-invertible Wilder arcs follows immediately from their classification. In 1963, an affirmative answer to question 1 was announced by Giffen [5] (a more detailed development of Giffen's ideas is given in [13]); however, Giffen developed no new invariant of local embedding type, so there was still no way of distinguishing nearly polyhedral arcs of the same penetration index. While S. J. Lomonaco ([8], 1967) succeeded in algebraically distinguishing the local types of arcs at one interior wild point, there was no theory of local embedding type for nearly polyhedral arcs.

To the best of the present author's knowledge, the invariants of local type of an oriented nearly polyhedral arc, developed in this paper and in [9], are the only ones available (other than penetration index invariants) that will differentiate one nearly polyhedral arc type