PACIFIC JOURNAL OF MATHEMATICS
Vol. 45, No. 2, 1973

THE AMALGAMATION PROPERTY IN EQUATIONAL CLASSES OF MODULAR LATTICES

G. GRÄTZER, B. JÓNSSON, AND H. LAKSER*

It is known that the class of all distributive lattices satisfies the Amalgamation Property. It will be shown that this is the only nontrivial equational class of modular lattices for which the Amalgamation Property holds. A second theorem gives further information about the Amalgamation Class of the class of all modular lattices, and of certain other equational classes.

1. Introduction. A class $\boldsymbol{K}$ of algebras (or in general, structures) is said to have the *Amalgamation Property* if for A, B_0, B_1 in $\boldsymbol{K}$ and for embeddings $f_i\colon A \rightarrow B_i$, $i = 0, 1$, there exist a C in $\boldsymbol{K}$ and embeddings $g_i\colon B_i \rightarrow C$, $i = 1, 2$, such that $f_0 g_0 = f_1 g_1$ (see Figure 1).

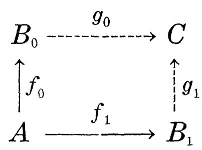

FIGURE 1.

For the history and importance of the Amalgamation Property we refer the reader to B. Jónsson [8] (see also G. Grätzer [5]).

The class of all lattices and the class of all distributive lattices both have the Amalgamation Property. The problem whether the class $\boldsymbol{M}$ of all modular lattices has the Amalgamation Property has been around for more than a decade.

In January of 1971 B. Jónsson announced [11] that $\boldsymbol{M}$ does not have the Amalgamation Property, in fact, any equational class $\boldsymbol{K}$ of modular lattices having the Amalgamation Property must satisfy the arguesian identity. This was followed by an announcement by G. Grätzer and H. Lakser [7] stating that every member of $\boldsymbol{K}$ can be embedded into the subspace lattice of an infinite dimensional projective geometry. Combining and extending these results, we can now prove the following:

THEOREM 1. *If an equational class* $\boldsymbol{K}$ *of modular lattices contains a nondistributive lattice, then* $\boldsymbol{K}$ *does not have the Amalgamation Property.*

If $\boldsymbol{K}$ does not have the Amalgamation Property, then we can use the concept of the Amalgamation Class of $\boldsymbol{K}$ (G. Grätzer and H. Lakser