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INDUCED TOPOLOGIES FOR QUASIGROUPS AND LOOPS

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The concept of a [semi] topological quasigroup is defined and the notions of induced groupoids and isotopes are extended to the topological case. Necessary and sufficient conditions are found in order for a continuously induced isotope of a semitopological quasigroup to be a semitopological quasigroup.

Given an injection i of a topological space $(A, \mathscr{A})$ into a set S acted on by a group, G, a topology $\mathscr{T}_A$ on S is introduced in a natural fashion under which i is continuous. When $S=Q$ is itself a semitopological quasigroup and G is generated by the left or right translations of Q the continuity or openness of i can be checked by comparing the topology $\mathscr{T}_A$ with that of Q. In particular this method is applied in § 3 to the study of topologically invariant subloops.

1. Preliminaries; continuously induced isotopes. We recall (cf. [2]) that a *quasigroup* $(Q, \cdot)$ is a closed binary system i.e., a *groupoid*, where for any $q \in Q$ the left and right translations λ_q, ρ_q, both define bijections on Q. Thus given a quasigroup $(Q, \cdot)$ we can define (cf. [2], [7]) the so called *conjugate operations* $/$, $\backslash$ on the set Q by $c/b = a$ iff $ab = c$ iff $a\backslash c = b$ for $a, b, c \in Q$. In the conjugate quasigroups $(Q, /)$ and $(Q, \backslash)$ we will denote the left and right translations determined by $q \in Q$ by $\lambda_q^/$, $\rho_q^/$, $\lambda_q^\backslash$, and $\rho_q^\backslash$ respectively. (In general, ρ_a^* will denote the right translation determined by a relative to the binary operation*.) A *loop* is a quasigroup with a two sided identity. A conjugate operation need not be associative; viz. if $G = (Z, +)$, where Z is the set of integers then $c/b = c - b$.

We now make the following definition (cf. [4], [5]):

DEFINITION 1.1. 1. A *semitopological quasigroup* [*loop*] is a triple $(Q, \cdot, \mathscr{T})$ where $(Q, \cdot)$ is a quasigroup [loop] and $(Q, \mathscr{T})$ a Hausdorff space in which the left and right (multiplications) translations, λ_a, ρ_b, $a, b \in Q$ are homeomorphisms of Q onto itself. We will write $Q = (Q, \cdot; \mathscr{T})$ is a s.t.q. [s.t.*l*.] in such cases.

2. A triple $(Q, \cdot; \mathscr{T})$ is a *topological quasigroup* [*loop*] if Q is a quasigroup [loop] and multiplication in Q and each of its conjugates is continuous in both variables with respect to the same topology $\mathscr{T}$. We will write Q is a t.q. [t.*l*.] in such cases.

One makes the obvious genralizations of a [*semi*] *topological group* (s.t.g.) ([5], p. 28) to [*semi*] *topological semigroup* (s.t.s.) and *groupoid* (s.t.gd.). We observe that a s.t.q. is homogeneous and that a s.t.g.