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TORSIONAL RIGIDITIES IN THE ELASTIC-PLASTIC TORSION OF SIMPLY CONNECTED CYLINDRICAL BARS

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Under elastic-plastic torsion, the circular shaft possesses the maximum resisting torque among all solid bars with the same cross-sectional area and the same angle of twist per unit length.

1. Introduction. Consider a simply connected cylindrical bar twisted by terminal couples. If the angle of twist per unit length is sufficiently small, then the bar behaves linearly elastic [18, 23, 26]. Under this circumstance, St. Venant succeeded in formulating it as a Neumann problem by means of his semi-inverse method, [22]. It was his contention that among all solid bars with the same cross-sectional area, the circular shaft gives the maximum torsional rigidity. This isoperimetric problem was first solved by Pòlya [15]. Later similar results have also been obtained for multiply connected cross-sections [17]. The results of Pòlya and Szegö have had much influence and further explorations of their problems have been continued up to the present time [4, 5, 13-17, 29].

According to the theory of plasticity [26, 27], if the angle of twist per unit length reaches a certain critical value, then some portion near the boundary of the bar becomes plastic. Moreover, the plastic region grows as the load increases, [26]. Although the elastic-plastic torsion problem has been stated quite precisely for a long time, [28], the answers to the basic existence and regularity problems are recent ones, [2, 9, 11, 12, 26]. However, before the elastic-plastic torsion problem was completely settled, Leavitt and Ungar already showed that the circular shaft is also the strongest one under completely plastic torsion, [10].

Since the elastic-plastic torsion problem can be so formulated that it includes both the purely elastic and the completely plastic torsions as special cases, [26], it is the objective of this note to present a proof for the statement in the Abstract. Needless to say that Pòlya's ideas in his first and third proofs of St. Venant's conjecture will play an essential role in this proof. On the other hand, the present theorem includes Pòlya's results as well as the one obtained in [10].

2. The elastic-plastic torsion problem. Denote by G the simply connected cross-section of a solid bar. We shall restrict G to have the following properties: (i) ∂G, boundary of G, possesses continuously varying curvature except at a finite number of corners,