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REAL PARTS OF UNIFORM ALGEBRAS

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This paper is concerned with identifying those uniform algebras B on $\Gamma = \{z: |z| = 1\}$ for which Re B—the space of real parts of the functions in B—equals Re A, where A denotes the disk algebra. It is shown that for any such algebra, there is an absolutely continuous homeomorphism Φ of Γ onto Γ so that $B = A(\Phi) = \{f(\Phi): f \in A\}$. A partial converse to this theorem also holds: If Φ is a homeomorphism of Γ onto itself which is of class C^2 with nowhere vanishing derivative, then Re $A(\Phi) =$ Re A.

For completeness, we recall that a uniform algebra on Γ is defined as a subalgebra of $C(\Gamma)$ which is closed in the norm $\|f\| = \max_\Gamma |f|$, contains the constants and separates the points of Γ. The disk algebra is the particular uniform algebra consisting of all functions in $C(\Gamma)$ which extend continuously to $\{z: |z| \leqq 1\}$ to be analytic on $D = \{z: |z| < 1\}$. Note that if Φ is a homeomorphism of Γ onto itself, then $A(\Phi)$ is a uniform algebra. In this paper we will frequently use the convention of writing $f(\theta)$ for $f(e^{i\theta})$ when f belongs to $C(\Gamma)$.

The first result along the lines discussed in this paper is due to Hoffman and Wermer [7]. They prove that if B is a uniform algebra on a compact Hausdorff space X such that Re B is closed in the norm of uniform convergence, then $B = C(X)$; in particular, the theorem holds if Re $B = C_r(X)$. A generalization of this fact by Sidney and Stout [10] says that if K is a closed subset of X and Re B_K is uniformly closed, then $B_K = C(K)$. Bernard [1, 2, 3] has provided extensions in a different direction. He shows that a Banach algebra $B \subseteqq C(X)$, with any norm, for which Re B is closed under uniform convergence must equal $C(X)$. Further, he gives some sufficient conditions on two Banach algebras $B_1 \subseteqq B_2 \subseteqq C(X)$, with Re $B_1 =$ Re B_2, to conclude that $B_1 = B_2$.

The theorems in this paper have previously been announced in Abstracts 71T-B10 and 71T-B94 of the *Notices of the American Mathematical Society*, 18 (1971). The author wishes to express his appreciation to Professor John Wermer for his many helpful suggestions and conversations during the research which led to these results.

II. Algebras on Γ with the same real parts.

THEOREM 1. *Let B be a uniform algebra on Γ with Re $B =$ Re A. Then there exists a homeomorphism Φ of Γ onto Γ so that $B = A(\Phi)$.*