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A VERY WEAK TOPOLOGY FOR THE MIKUSINSKI FIELD OF OPERATORS

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Using a generalized Laplace transformation the Mikusinski field is given a topology T such that sequences which converge in the sense of Mikusinski converge with respect to T, such that the mapping $q \to q^{-1}$ is continuous and such that the series $\sum (-\lambda)^n s^n/n!$ converges to the translation operator $e^{-\lambda s}$.

In [3] it is shown that the notion of convergence defined in [8] for the Mikusinski field of operators is not topological. Topologies for the Mikusinski field are given in [1], [3], and [9]. In the present paper we endow this field with a topology T such that sequences which converge in the sense of Mikusinski converge with respect to T, such that the identity

$$e^{-\lambda s} = \sum_{k=0}^{\infty} \frac{(-\lambda)^k}{k!} s^k \qquad (\lambda > 0)$$

holds and such that the mapping $q \to q^{-1}$ is continuous. The author wishes to acknowledge that this paper constitutes proofs of assertions proposed by Gregers Krabbe [7].

Let L denote the family of complex-valued functions which are locally integrable on $[0, \infty)$. Under addition and convolution L is an integral domain. If Q denotes the quotient field of L then Q is the Mikusinski field of operators. Elements of Q will be denoted $\{f(t)\}$: $\{g(t)\}$ and the injection of L into Q will be denoted $f \to \{f(t)\}$. We define S to be the set of all f in L for which the integral

$$\int_0^\infty e^{-zt} f(t)dt$$

converges for some z. For f in S let

$$\bar{f}(z) = \int_0^\infty e^{-zt} f(t)dt$$

and $\bar{S} = \{\bar{f} : f \in S\}$. Each element of $\bar{S}$ is holomorphic in some right half-plane. Let B denote the set of all sequences $(\bar{f}_n)$ of nonzero elements of $\bar{S}$ for which there exists f in L such that

$$(1) \qquad f_n = f \text{ on } (0, n) \qquad \text{for all } n .$$

For a given f the set of all elements of B satisfying (1) will be