PACIFIC JOURNAL OF MATHEMATICS
Vol. 47, No. 1, 1973

ABEL-ERGODIC THEOREMS FOR SUBSEQUENCES

RYOTARO SATO

Let T be a positive linear contraction on an L^1-space and $k_1, k_2, \cdots$ an increasing sequence of positive integers. In this paper the almost everywhere convergence of Abel averages $\sum_{i=1}^{\infty} r^{k_i} T^{k_i} f / \sum_{i=1}^{\infty} r^{k_i}$ for the sequence $k_1, k_2, \cdots$ as $r \uparrow 1$ is investigated.

In [3], A. Brunel and M. Keane defined uniform sequences for increasing sequences of positive integers and proved that if ϕ is a measure preserving transformation on a finite measure space then for any uniform sequence $k_1, k_2, \cdots$ and for any integrable function f, Cesàro averages of $f(\phi^{k_i} \cdot)$ converge almost everywhere. The author [13], [14] has recently generalized and extended this result to one at the operator theoretic level. On the other hand, the work of G.-C. Rota [11] suggests that it would be of interest to consider the almost everywhere convergence of Abel averages for uniform sequences. These are the starting points for the study in this paper.

2. **Main results.** Let $(\Omega, \mathscr{B}, m)$ be a σ-finite measure space with positive measure m and $L^p(\Omega) = L^p(\Omega, \mathscr{B}, m)$, $1 \leqq p \leqq \infty$, the usual (complex) Banach spaces. Let T be a *positive* linear operator from $L^1(\Omega)$ to $L^1(\Omega)$ with $\|T\|_1 \leqq 1$. We shall say that *the Abel-individual ergodic theorem holds for T* if for any uniform sequence $k_1, k_2, \cdots$ (for the definition, see [3]) and for any $f \in L^1(\Omega)$, the limit

$$\tilde{f}(\omega) = \lim_{r \uparrow 1} \frac{\sum_{i=1}^{\infty} r^{k_i} T^{k_i} f(\omega)}{\sum_{i=1}^{\infty} r^{k_i}}$$

exists almost everywhere and $\tilde{f} \in L^1(\Omega)$. The main results of this paper are the following two theorems.

THEOREM 1. *If T maps, in addition, $L^p(\Omega)$ into $L^p(\Omega)$ for some p with $1 < p < \infty$ and $\|T\|_p \leqq 1$, then the Abel-individual ergodic theorem holds for T.*

THEOREM 2. *If there exists a strictly positive function $h \in L^1(\Omega)$ such that the set*

$$\left\{(1 - r) \sum_{k=0}^{\infty} r^k T^k h;\ 0 < r < 1\right\}$$

is weakly sequentially compact in $L^1(\Omega)$, then the Abel-individual ergodic theorem holds for T.