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ON THE CLASSIFICATION OF LINDENSTRAUSS SPACES

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A Lindenstrauss space is a real Banach space X such that its dual X^* is linearly isometric to $L_1(\mu)$ for some measure μ. The purpose of this paper is to describe how certain classical types of Lindenstrauss spaces are characterized by mappings from a compact Hausdorff space S into $C(S)^*$.

Let S be a compact Hausdorff space and $\rho: S \rightarrow C(S)^*$ a bounded function such that for each $f \in C(S)$, the function f_ρ defined by $f_\rho(s) = \int f d\rho(s)$ is integrable with respect to each regular Borel measure on S. Thus ρ induces a natural bounded linear operator P on $C(S)^*$ defined by $(P\mu)(f) = \int f_\rho d\mu$ for all $\mu \in C(S)^*$ and $f \in C(S)$. If (i) $\|\rho(s)\| \leqq 1$ for all $s \in S$, and (ii) whenever $\mu \in C(S)$ and $\int f d\mu = 0$ for all $f \in C(S)$ with $f = f_\rho$, then $\int f_\rho d\mu = 0$ for all $f \in C(S)$, then ρ is said to be an *affine mapping.*

It was shown in [1] that if ρ is affine, then $X_\rho = \{f \in C(S): f = f_\rho\}$ is a Lindenstrauss space, P is a contractive projection on $C(S)^*$ with kernel equal to $X_\rho^\perp = \{\mu \in C(S)^*: \mu(f) = 0 \text{ for all } f \in X_\rho\}$. Moreover, the restriction mapping $\mu \rightarrow \mu \mid X_\rho$ is a linear isometry from the range of P onto X_ρ^*.

Condition (ii) of the definition of an affine mapping is usually the hardest to verify. In [7] Gleit gives a nontrivial example when $\rho(s) \geqq 0$ for all $s \in S$. Although he did not actually use this terminology, careful inspection of his proof yields that the mapping he postulates is indeed an affine mapping. In fact, slight modifications in his proof yields the following general result.

THEOREM 1. (*Gleit*). *Let S be a compact Hausdorff space and T a closed subset of S. Let $\rho: S \rightarrow C(S)^*$ be a mapping such that*

(a) *$\|\rho(s)\| \leqq 1$ for all $s \in S$,*

(b) *$\rho(s) = \varepsilon_s$ (i.e., point evaluation) for $s \in S\backslash T$,*

(c) *for $T_2 = \{s \in T: \rho(s) = \varepsilon_s\}$ and $T_1 = T\backslash T_2$, we suppose that $T_1 \neq S$ and $|\rho(s)|(T_1) = 0$ for all $s \in T_1$,*

(d) *$\rho \mid T$ is weak* continuous.*

Then ρ is an affine mapping and X_ρ^* is, in fact, linearly isometric to $\{\mu \in C(S)^*: |\mu|(T_1) = 0\}$.

As mentioned above, Gleit assumes in addition that $\rho(s) \geqq 0$ for