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THE PRODUCT OF F-SPACES WITH P-SPACES

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A condition on a basically disconnected space X is known which is necessary and sufficient for the product space $X \times Y$ to be basically disconnected for every P-space Y. This same condition, when applied to an F'-space X, guarantees that $X \times Y$ is an F'-space whenever Y is a P-space and is necessary for this result. The principal result of this paper establishes that this condition is not sufficient when applied to F-spaces. A condition which is sufficient but not necessary is also derived.

1. Introduction. The notation and general point of view are those of the Gillman and Jerison textbook [5]. In particular, all hypothesized spaces are completely regular Hausdorff. The reader should recall from [4] the following characterizations. A space X is: a P-space if and only if each cozero set is closed; a basically disconnected space if and only if each cozero set has open closure; a U-space if and only if disjoint cozero sets can be separated by an open-and-closed set; an F-space if and only if disjoint cozero sets can be completely separated; and an F'-space if and only if disjoint cozero sets have disjoint closures. It is clear from these characterizations that the conditions named grow progressively weaker.

In [3] Gillman asked for a necessary and sufficient condition that a product of two spaces be an F-space and, parenthetically, for a necessary and sufficient condition that a product of two spaces be a basically disconnected space. Curtis had shown [2] that if $X \times Y$ is an F'-space then either X or Y must be a P-space. It is easily seen that if $X \times Y$ has any of the properties listed above so must both X and Y for X and Y nonempty. Observing also that the product of a space X with a discrete space Y has any of the above mentioned properties which X has, one can rephrase the question in the form: For which spaces X with property A does the product $X \times Y$ have property A for every P-space Y?

This question was answered for the properties F' and basically disconnected in [1]. The condition was that the space be countably locally weakly Lindelöf (appreviated CLWL). That is, for every countable collection $\{\Gamma_n\}_{n=1}^{\infty}$ of open covers of X and each point x of X there must be a neighborhood V of x and, for each n, a countable subfamily Δ_n of Γ_n such that $V \subseteqq \mathrm{cl} \bigcup \Delta_n$.

Since F-spaces are F'-spaces the condition that X be CLWL is