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MAPPING SPACES AND *CS*-NETWORKS

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In this paper the space of maps from an $\aleph_0$-space to a space Y is studied by means of convergent sequence-networks. The notion of a *cs-σ*-space, a simultaneous generalization of metric spaces and $\aleph_0$-spaces, is defined, and it is shown that if Y is a (paracompact) *cs-σ*-space then the mapping space from X to Y is a (paracompact) *cs-σ*-space when equipped with either the compact-open or the *cs*-open topology. It is proved that the compact sets are the same in the two topologies. The class of *cs-σ*-spaces and the class of $\aleph$-spaces introduced by O'Meara are shown to be identical in the presence of paracompactness.

In this paper all maps are continuous and all spaces Hausdorff.

1. *CS*-networks. We shall call a collection $\mathscr{P}$ of subsets of a space X a *k-network* for X if whenever $C \subset U$, with C compact and U open in X, there exist finitely many elements of $\mathscr{P}$ whose union covers C and lies in U. This is a slight modification of what E. Michael [2] called a *pseudobase*. We may define the *$\aleph_0$-spaces* of Michael as regular spaces with a countable k-network.

If X is a space with topology $\mathscr{T}$ we shall denote by $k(X)$ the k-space obtained by retopologizing X so that a set is closed if its intersection with every $\mathscr{T}$-compact set is $\mathscr{T}$-closed.

If $\{z_1, z_2, \cdots\}$ is a sequence of points which converges to a point z, then we call the set $Z = \{z, z_1, z_2, \cdots\}$ a *convergent sequence* and denote by Z_n the convergent sequence $\{z, z_n, z_{n+1}, \cdots\}$.

A collection $\mathscr{P}$ of subsets of a space X is a *convergent sequence-network* or, more conveniently, a *cs-network* for X if whenever $Z \subset U$, with Z a convergent sequence and U open in X, then $Z_n \subset P \subset U$ for some n and some $P \in \mathscr{P}$. We call a collection $\mathscr{P}$ of subsets of X a *network* for X if whenever $x \in U$ with U open in X, then $x \in P \subset U$ for some $P \in \mathscr{P}$.

The notion of cs-network was introduced in [1] where the following theorem was proved.

THEOREM 1. *For a topological space X the following are equivalent:*
(1) *X is an $\aleph_0$-space.*
(2) *X is a regular space with a countable cs-network.*

We shall call a regular space with a σ-locally finite cs-network a *cs-σ-space*. It is clear from Theorem 1 that every $\aleph_0$-space is a *cs*-