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THE BEHAVIOR OF THE NORM OF AN AUTOMORPHISM OF THE UNIT DISK

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For $f(z)$ analytic on the closed unit disk,

$$f(z) = \sum a_k z^k ,$$

let

$$\|f\| = \sum |a_k| .$$

In this paper the following result is obtained: Theorem. Let $f(z)$ be an automorphism of the unit disk:

$$f(z) = e^{i\zeta}\frac{z-\alpha}{1-\bar{\alpha}z}, 0 < |\alpha| < 1, \zeta \text{ real} .$$

Then

$$\frac{1}{\sqrt{n}}\|f^n\| \sim \frac{8\sqrt{2}}{\Gamma^2\left(\frac{1}{4}\right)}(1-\beta^2)^{1/2}F\left(\frac{1}{2}, \frac{3}{4}; \frac{3}{2}; 1-\beta^2\right)$$

as $n \to \infty$ where $F = {}_2F_1$ is the hypergeometric function and

$$\beta = \frac{1-|\alpha|}{1+|\alpha|} .$$

1. Introduction. In a more general context we denote by A the class of all functions with absolutely convergent Fourier series and define

$$\|f\|_A = \sum_{-\infty<k<\infty} |\hat{f}(k)| .$$

If $f(z)$ is analytic on the closed unit disk, then $f(e^{it}) \in A$ and $\|f\| = \|f\|_A$.

The asymptotic behavior of $\|f^n\|_A$ has been studied in several recent papers. Kahane [5] has shown that if f is real, analytic, periodic of period 2π, and nonconstant, then there exist two positive constants C_1 and C_2 such that $C_1\sqrt{n} < \|e^{inf}\|_A < C_2\sqrt{n}$. More recently in [3] the behavior of $\|f^n\|_A$ has been studied in the case where $f \in A$, $|f(t)| \leqq 1$ and $|f(t)| = 1$ for at most a finite number of points in $[0, 2\pi]$. Further results and connections with summability methods, the stability of difference schemes and the structure theory of A may be found, respectively, in [3], [4] and the recent monograph by Kahane [6].

2. Preliminary lemmas. In this section we give several results