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A NEW LOOK AT SOME FAMILIAR SPACES OF INTERTWINING OPERATORS

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When T_1 and T_2 are bounded operators on a Hilbert space, solutions to the equation $XT_1 = T_2X$ are called intertwining operators for T_1 and T_2. Several familiar spaces of intertwining operators are examined and a new method is proposed for studying the duality relationship that frequently exists between the space of intertwining operators and its subspace of compact intertwining operators.

Evidently such operators form a Banach space if the usual operator norm is employed. We denote this space by $\mathscr{I}(T_1, T_2)$.

Such spaces of intertwining operators are abundantly present in the literature of operator theory. We cite these examples:

EXAMPLE 1. If D is a diagonal operator with distinct, nonzero diagonal entries, then $\mathscr{I}(D, D)$, the commutant of D, is simply the familiar space of all diagonal matrices and is isomorphic to ℓ^∞.

EXAMPLE 2. In case S is either the simple unilateral or bilateral shift, $\mathscr{I}(S, S)$ is the space of analytic Toeplitz operators or Laurent operators respectively.

EXAMPLE 3. Let T be the restriction of the adjoint of the simple shift to an invariant subspace. In [5], Kriete, Moore, and Page studied $\mathscr{I}(T^*, T)$.

EXAMPLE 4. If S is a unilateral shift (of any multiplicity), then $\mathscr{I}(S, S^*)$ is the space of all S-Hankel operators.

All of the above spaces of intertwining operators have been studied in depth. The phenomenon which motivates our investigation is that in Examples 1, 3, and 4 the space of intertwining operators in question is isomorphic to the second dual space of its compact part, i.e., to the second dual of the space obtained by intersecting $\mathscr{I}(T_1, T_2)$ with the space of compact operators on $\mathscr{H}$.

By way of contrast, there are no compact operators at all in the spaces of Example 2.

That the spaces of Examples 1, 3, and 4 above have the stated biduality property is seen by representing the spaces of intertwining operators under study as familiar function spaces (or quotient spaces of such), and then determining the duality properties of said function