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STRUCTURE HYPERGROUPS FOR MEASURE ALGEBRAS

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An abstract measure algebra A is a Banach algebra of measures on a locally compact Hausdorff space X such that the set of probability measures in A is mapped into itself under multiplication, and if μ is a finite regular Borel measure on X and $\mu << \nu \in A$ then $\mu \in A$. If A is commutative then the spectrum of A, Δ_A, is a subset of the dual of A, A^*, which is a commutative W^*-algebra. In this paper conditions are given which insure that the weak-* closed convex hull of Δ_A, or of some subset of Δ_A, is a subsemigroup of the unit ball of A^*. This statement implies the existence of certain byper-group structures. An example is given for which the conditions fail.

The theory is then applied to the measure algebra of a compact P^*-hypergroup, for example, the algebra of central measures on a compact group, or the algebra of measures on certain homogeneous spaces. A further hypothesis, which is satisfied by the algebra of measures given by ultraspherical series, is given and it is used to give a complete description of the spectrum and the idempotents in this case.

A hypergroup is a locally compact space on which the space of finite regular Borel measures has a commutative convolution structure preserving the probability measures. The spectrum of the measure algebra of a locally compact abelian group is the semigroup of all continuous semicharacters of a commutative compact topological semigroup (Taylor [7], or see [2, Ch. 1]). In this paper we consider the spectrum of an abstract measure algebra and investigate the question of whether the spectrum or some subset of it has a hypergroup structure.

Section 1 of the paper contains a general theorem on the existence of hypergroup structures on the spectrum of an abstract measure algebra. The fact that the dual space of an appropriate space of measures is a commutative W^*-algebra is of basic importance in the proof of this theorem. This section also contains an example of a compact hypergroup whose measure algebra does not satisfy the hypotheses of the theorem.

In §2 we recall the definition of a compact P^*-hypergroup from a previous paper [1] and apply the main theorem of §1 to this situation. The result is that the closure of the set of characters of the hypergroup in the spectrum is a compact semitopological hypergroup and is a set of characters on another compact semitopological hypergroup.