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NONSOLVABLE FINITE GROUPS ALL OF WHOSE LOCAL SUBGROUPS ARE SOLVABLE, IV

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In this paper, the simple N-groups are classified for which $e \geqq 3$ and $2 \in \pi_4$. This latter condition means that a Sylow 2-subgroup contains a normal elementary abelian subgroup of order 8 and does not normalize any nonidentity odd order subgroup.

As in III, the proofs rely heavily on the fact that many subgroups of odd order are contained in just one maximal subgroup. The numbering of the sections is a continuation of III. The bibliographical references are to be found at the end of I. The predecessors to this paper are: Nonsolvable Finite Groups all of whose Local Subgroups are solvable, I, II, III: Bull A. M. S., 74 (1968), 383-437; Pacific J. Math., 33 (1970), 451-536; Pacific J. Math., 39 (1971), 483-534.

13. The case $2 \varepsilon \pi_4$; first reduction.

THEOREM 13.1.

(a) *If $p \in \pi_3$, then $\mathscr{A}(p) \subseteq \mathscr{M}^*(\mathfrak{G})$. ($\mathscr{A}(p)$ is defined in Definition 2.10, and $\mathscr{M}^*(\mathfrak{G})$ is defined in Definition 2.7.)*

(b) *If $p \in \pi_3$, $\mathfrak{P}$ is a S_p-subgroup of $\mathfrak{G}$ and $\mathfrak{M}$ is the unique element of $\mathscr{M}\mathscr{S}(\mathfrak{G})$ which contains $\mathfrak{P}$, then*

(i) $\mathfrak{P} \subseteq \mathfrak{M}'$,

(ii) *for each G in $\mathfrak{G} - \mathfrak{M}$, S_p-subgroups of $\mathfrak{M} \cap \mathfrak{M}^G$ are of order 1 or p.*

Proof. Theorem 10.7 implies (a); (b)(i) is a consequence of (a) and a standard transfer theorem; (b)(ii) can be established by imitating the proof of Theorem 0.25.6.

LEMMA 13.1. *If $\mathfrak{X}$ is non identity 2-subgroup of $\mathfrak{G}$, then $O_{2'}(N(\mathfrak{X})) = 1$.*

Proof. Set $\mathfrak{N} = N(\mathfrak{X})$ and let $\mathfrak{T}$ be a S_2-subgroup of $\mathfrak{N}$. Suppose by way of contradiction that $O_{2'}(\mathfrak{N}) \neq 1$. First, suppose $|\mathfrak{X}| = 2$. Let $\mathfrak{P}$ be a minimal normal subgroup of $\mathfrak{N}$ of odd order. Thus, $\mathfrak{P}$ is a p-group for some odd prime p. Let $\mathfrak{H}$ be a maximal 2, p-subgroup of $\mathfrak{G}$ which contains $\mathfrak{T}\mathfrak{P}$. Let $\mathfrak{H}_2$, $\mathfrak{H}_p$ be a Sylow system of $\mathfrak{H}$ with $\mathfrak{T} \subseteq \mathfrak{H}_2$, $\mathfrak{P} \subseteq \mathfrak{H}_p$.

First, suppose $O_p(\mathfrak{H}) = 1$. Let $\mathfrak{R} = O_2(\mathfrak{H})$. By the $\mathfrak{P} \times \mathfrak{Q}$-lemma,