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VALUE DISTRIBUTION OF LINEAR COMBINATIONS OF AXISYMMETRIC HARMONIC POLYNOMIALS AND THEIR DERIVATIVES

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In this paper the geometry of the value distribution of linear combinations of axisymmetric harmonic polynomials (AHP) and their derivatives is studied using the Bergman integral operator method and methods from the analytic theory of polynomials. For a given AHP, zero free cones in E^3 can be determined which are stationary for specified classes of these linear combinations in the sense that the given AHP describes cones which have an empty intersection with the level sets of all linear combinations from each class.

In addition to these results, an AHP analog to the classical theorem of Lucas' is obtained. The above results are extended to generalized axisymmetric harmonic polynomials by an operator due to R. P. Gilbert.

The study of the value distribution of AHP by the Bergman method was initiated by Morris Marden [4]. In that paper, the Bergman method [1] was used to transform polynomials of one complex variable into AHP. Methods from the analytic theory of polynomials were used with operator to obtain theorems on the location of sets in E^3 where an AHP omits a given complex value. These results were specified in terms of a pair of cones in E^3 which were functions of the convex hull of the zero set of a polynomial of one complex variable *associated* with the AHP by the Bergman operator.

Let (x, y, z) be rectangular coordinates and (x, ρ, ϕ) be cylindrical coordinates with

$$\rho^2 = y^2 + z^2,\ y = \rho\cos\phi,\ z = \rho\sin\phi\ .$$

A function is said to be axisymmetric if it is independent of ϕ.

Every AHP can be represented by the Bergman method as the integral transform of a polynomial of one complex variable (see [4]) which shall be referred to as the *associate* of the AHP. That is, if $H(x, \rho)$ is an AHP then there is a unique polynomial $h(\zeta)$ of the complex variable ζ such that

$$H(x, \rho) = \frac{1}{2\pi}\int_0^{2\pi} h(x + i\rho\cos t)dt\ .$$

In addition, each AHP can be represented in the form (see [3])