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SUBMANIFOLDS OF ACYCLIC 3-MANIFOLDS

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It is proved that, from the viewpoint of "geometric" homology theory, an arbitrary embedding of a closed surface S in any 3-manifold with trivial first homology group looks exactly like the standard embedding of S in the euclidean 3-space. A consequence: every compact subset of a 3-manifold with trivial first homology group can be embedded in a homology 3-sphere. Necessary and sufficient (homological) conditions are given for a compact 3-manifold to be embeddable in some acyclic 3-manifold (or in some homology 3-sphere).

1. Definitions and preliminaries.

Manifolds. We work in the PL category. Each manifold is supposed to have a fixed PL structure. If M is a manifold, then by a submanifold of M or by a surface, simple closed curve, arc, etc., in M we always mean a respective object contained in M as a subpolyhedron (in the chosen PL structure of M). All maps are assumed to be PL. Our manifolds are never automatically assumed to be without boundary, compact, connected, or orientable. However, by a *surface* we mean a compact, connected, orientable 2-manifold. A *cube with n handles* is a 3-manifold homeomorphic to a regular neighborhood of a connected finite linear graph of Euler characteristic $1 - n$ in E^3.

We denote the interior of a manifold M by $\operatorname{int} M$ and the boundary by $\operatorname{Bd} M$. However, if M is oriented, then by ∂M we denote the manifold $\operatorname{Bd} M$ oriented coherently with M. The symbol ∂ also denotes the boundary in the homological sense. Let M be an oriented manifold and P a codimension 0 submanifold of M. Whenever we talk of P as an oriented manifold, we assume that P has the orientation inherited from M, unless explicitly stated otherwise. If M is an oriented manifold, then M with the opposite orientation is sometimes denoted by $-M$.

Homology. All homology and cohomology groups, cycles, chains, etc., have integer coefficients. If z_1, z_2 are n-cycles in a space X, then $z_1 \sim z_2$ means "z_1 is homologous to z_2". A compact oriented n-submanifold N of an m-manifold M generates a uniquely determined PL n-chain in M. This chain is a cycle if and only if N is a closed manifold. We shall make no distinction in notation between N and the n-chain it represents. If M is a manifold of dimension