PACIFIC JOURNAL OF MATHEMATICS
Vol. 49, No. 1, 1973

SOME COMMUTANTS IN $B(c)$ WHICH ARE ALMOST MATRICES

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We determine necessary and sufficient conditions for two linear operators in $B(c)$ to commute. Specializing one of the operators to be a conservative triangular matrix we determine that most such operators have commutants consisting of almost matrices of a special form.

Almost matrices were developed in [10] for reasons not related to this paper, but they find application here in that the commutants in $B(c)$ of certain matrices must be almost matrices.

Let c denote the space of convergent sequences, $B(c)$ the algebra of all bounded linear operators over c, e the sequence of all ones, and e^k the coordinate sequences with a one in the kth position and zeros elsewhere. If $T \in B(c)$, then one can define continuous linear functionals χ and χ_i by $\chi(T) = \lim Te - \sum_k \lim (Te^k)$ and $\chi_i(T) = (Te)_i - \sum_k (Te^k)_i$, $i = 1, 2, \cdots$. (See, e.g. [9, p. 241].) It is known [1, p. 8] that any $T \in B(c)$ has the representation $T = v \otimes \lim + B$, where B is the matrix representation of the restriction of T to c_0, the subspace of null sequences, v is the bounded sequence $v = \{\chi_i(T)\}$, and $v \otimes \lim x = (\lim x)v$ for each $x \in c$.

The second adjoint of T (see, e.g. [1, p. 8] or [10, p. 357]) has the matrix representation

$$(*) \qquad T'' = \begin{pmatrix} \chi(T) & b_1 & b_2 & \cdot & \cdot & \cdot \\ \chi_1(T) & b_{11} & b_{12} & \cdot & \cdot & \cdot \\ \chi_2(T) & b_{21} & b_{22} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

where the b_i's occur in the representation of $\lim \circ T \in c'$ as $(\lim \circ T)(x) = \lim (Tx) = (T) \lim x + \sum_k b_k x_k$; namely, $b_i = \lim Te^i$. With the use of $(*)$ it is easy to describe the commutant of any $Q \in B(c)$.

THEOREM 1. *Let* $Q = u \otimes \lim + A \in B(c)$. *Then* Com (Q) *in* $B(c) = \{T = v \otimes \lim + B \in B(c)$: *$T$ satisfies* (1)–(3)$\}$, *where*

$$(1) \qquad u_n \chi(T) + \sum_{k=1}^{\infty} a_{nk} v_k = v_n \chi(Q) + \sum_{k=1}^{\infty} b_{nk} u_k \, ; \qquad n = 1, 2, \cdots$$

$$(2) \qquad u_n b_k + \sum_{j=1}^{\infty} a_{nj} b_{jk} = v_n a_k + \sum_{j=1}^{\infty} b_{nj} a_{jk} \, ; \qquad n, k = 1, 2, \cdots$$