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CONVERGENCE OF BAIRE MEASURES

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Assume that there are no measurable cardinals. Then E. Granirer has proved that if a net $\{m_i\}$ of finite Baire measures on a completely regular Hausdorff space converges weakly to a finite Baire measure m, then $\{m_i\}$ converges to m uniformly on each uniformly bounded, equicontinuous subset of C^b, the space of bounded continuous functions. In this paper a relatively simple proof of Granirer's theorem is given based on a recent result of the author. The same method is used to prove the following analogue of Granirer's theorem. Let $\{m_i\}$ be a net of Baire measures on X each having compact support in the realcompactification of the underlying space X, and assume that $\int_X fdm_i \to \int_X fdm$ for every continuous function f on X where m is a Baire measure having compact support in the realcompactification of X. Then $\{m_i\}$ converges to m uniformly on each pointwise bounded, equicontinuous subset of C, the space of continuous functions on X. (The situation in the presence of measurable cardinals is also treated.)

In what follows, X will denote a completely regular Hausdorff space, C will denote the linear space of all continuous real-valued functions on X and C^b will denote the subspace of C consisting of all the uniformly bounded functions in C. The *Baire algebra* is the smallest σ-algebra on X with respect to which each of the functions in C is measurable. (Equivalently, it is the σ-algebra generated by the zero sets in X.) The linear space of all signed Baire measures on X with finite variation is denoted by M_σ, and the set of nonnegative elements in M_σ (i.e., the set of finite Baire measures) is denoted by M_σ^+. The space M_σ and C^b may be paired in the sense of Bourbaki by the bilinear form $\langle m, f\rangle = \int_X fdm = \int_X fdm^+ - \int_X fdm^-$ for all $m \in M_\sigma$ and all $f \in C^b$. By the *weak topology* on M_σ, will we mean the topology $\sigma(M_\sigma, C^b)$.

Let νX denote the realcompactification of X. (See [2], p. 116.) A Baire measure m on X is said to have *compact support in the realcompactification of* X if there is a compact set $G \subset \nu X$ such that for every zero set Z in νX with $G \subset Z$, it follows that $m(X \cap Z) = m(X)$. Let M_c denote the subspace of M_σ consisting of those elements whose total variations have compact support in the realcompactification of X. The set of nonnegative elements of M_c is denoted by M_c^+. It is not hard to verify that if $m \in M_c^+$, then $C \subset L^1(m)$. Hence the