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THE SELF-EQUIVALENCES OF AN H-SPACE

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This paper studies the group $E(X)$ of self-homotopy-equivalences of a space X. Under mild (necessary) restrictions, it is shown that if X is an H-space then $E(X)$ is both finitely presented and Hopfian.

This paper studies the group of self-equivalences of a CW-complex X. This group, denoted by $E(X)$, is formed by taking the homotopy classes of homotopy equivalences from X to itself, and using composition as the group operation. Thus, categorically, $E(X)$ is the homotopy analog of an automorphism group. This group is important in topology because of its connection with the general problem of finding a complete set of homotopy invariants. It is known that a Postnikov system, in general, over-determines the homotopy type of a space. This happens because of the choices involved in picking the Postnikov invariants. $E(X)$ measures the indeterminacy that arises in this situation.

In addition, knowledge about $E(X)$ is related to the construction of classifying spaces. Let $LF(B)$ denote the fiber homotopy equivalence classes of Hurewicz fibrations over B with fibers the homotopy type of F; $H(F)$ denote the space of homotopy equivalences of F; and $B_{H(F)}$ denote the Dold-Lashof classifying space of $H(F)$. Then the space $B_{H(F)}$ represents the functor $LF(-)$. Since $E(F) = \Pi_1(B_{H(F)})$, knowledge about $E(F)$, such as whether or not it is finitely generated or presented, is of importance.

Previous investigations of the group $E(X)$ have been made from a general point of view in [2], [3], [10], [11], [16], and [17]. However, despite the extensive literature that exists, very little is known about this group and its properties. In particular, it is not known if $E(X)$ is finitely generated for finite complexes (in general, it is an infinite non-abelian group). W. Shih has claimed that, for finite complexes, $E(X)$ is finitely generated [2, p. 295]. However, no details of his work have appeared, and we have found objections to his results [18]. The finite generation question is regarded as open.

In studying $E(X)$, there is a natural restriction to place on the space X being considered. In this paper *it is always assumed that X is either finite-dimensional or has only finitely many nonzero homotopy groups*. Without one of these restrictions there are obvious counterexamples to the finite generation of $E(X)$. In addition, *it is always assumed that X is simply connected*. Modulo these restrictions, one hopes to show that: