PACIFIC JOURNAL OF MATHEMATICS
Vol. 50, No. 2, 1974

ON METRIZABILITY OF COMPLETE MOORE SPACES

G. M. REED

This paper is concerned with the relationships between certain 'strong' completeness properties in Moore spaces and with conditions under which Moore spaces satisfying these properties are metrizable.

In [3], Heath showed that each regular T_2-space which admits a strongly complete semi-metric is a complete Moore space. Furthermore, in [6], Heath established that each separable Moore space which admits a strongly complete semi-metric is metrizable. In [10], the author defined strong star-screenability, a property shared by separable spaces and metrizable spaces, and conjectured that separability could be replaced by strong star-screenability in Heath's result. In this paper, the author establishes relationships between different types of completeness in Moore spaces and gives two new metrization theorems for complete Moore spaces. From these results, it follows that each strongly star-screenable Moore space which admits a continuous, strongly complete semi-metric is metrizable.

An admissible semi-metric d for a T_2-space S is a distance function for S such that (1) if each of x and y is a point of S, then $d(x, y) = d(y, x) \geqq 0$, (2) $d(x, y) = 0$ if and only if $x = y$, and (3) the topology of S is invariant with respect to d. A semi-metric d for the space S is said to be strongly complete provided that if $M_1, M_2, \cdots$ is a decreasing sequence of closed sets such that for each i, $M_i \subset \{y \in S \mid d(x_i, y) < 1/i\}$ for some $x_i \in S$, then $\bigcap M_i \neq \emptyset$. A space which admits a strongly complete semi-metric is said to be strongly complete. A development for a space S is a sequence $G_1, G_2, \cdots$ of open coverings of S such that, for each n, G_{n+1} is a subcollection of G_n, and for each point p and each open set D containing p, there is an integer n such that every element of G_n containing p is a subset of D. A development $G_1, G_2, \cdots$ for the space S is said to be complete (sequentially complete) provided that if $M_1, M_2, \cdots$ is a monotonic sequence of closed sets such that for each i, $M_i \subset g_i$ for some $g_i \in G_i$ ($M_i \subset \mathrm{st}(x_i, G_i)$ for some $x_i \in S$), then $\bigcap M_i \neq \emptyset$. A regular space having a development is a Moore space [1]. A Moore space having a complete (sequentially complete) development is said to be complete (sequentially complete). The property of sequential completeness is due to Traylor in [11]. Although each of strong completeness and sequential completeness is stronger than completeness in Moore spaces ([8] and [11]), for pointwise paracompact Moore spaces, all three are equivalent ([4] and [11]). A space S is said to be star-screenable