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THE LOCAL COMPACTNESS OF υX

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Necessary and sufficient conditions are given for the local compactness of the Hewitt realcompactification υX of a completely regular Hausdorff space X; the conditions are expressed in terms of the space X alone. In addition, the local compactness of other extensions is considered.

Introduction. There has been much recent interest in determining conditions on a completely regular Hausdorff space X that are equivalent to the local compactness of its Hewitt realcompactification υX. This interest stems primarily from the fact that the seemingly artificial hypothesis "υX is locally compact" enters quite naturally into the examination of the relation $\upsilon X \times \upsilon Y = \upsilon(X \times Y)$. Apparently the only known condition equivalent to the local compactness of υX is one discussed by Comfort in [1] and [2]. As remarked by Comfort, the condition is not on X alone, but involves υX essentially in its statement.

In the present paper a condition on X is given which is equivalent to the local compactness of υX (Theorem 2.7) and a number of known results are obtained as corollaries of this characterization theorem. Another characterization (Theorem 2.3) is given of the local compactness of υX in terms of real maximal ideals.

It was shown by Comfort in [1] and [2] that the local pseudocompactness of X plays an important role in connection with the local compactness of υX. The precise role is established below, where it is shown that the local pseudocompactness of X is equivalent to the local compactness of the extension ηX of X constructed by Johnson and Mandelker in [9]. In addition a characterization is given of those spaces for which the extension ψX constructed by Johnson and Mandelker is locally compact.

Our attention will be restricted entirely to completely regular Hausdorff spaces. The terminology and notation of [4] will be used without further comment.

Given $f \in C(X)$ the symbols $N(f)$ and $S(f)$ represent respectively $\{x \in X: f(x) \neq 0\}$ and $\text{cl}_X \{x \in X: f(x) \neq 0\}$; these sets are called the *cozero set* and the *support* of f. If A and B are subsets of X, write $A \ll B$ if A is completely separated from $X - B$. We shall frequently apply [4, 1.15] to construct additional separating zero sets when $A \ll B$.

The symbol $\boldsymbol{M}^p$ will denote the maximal ideal in $C(X)$ which corresponds to the point p of βX, and $\mathscr{M}^p$ will denote the corresponding z-ultrafilter (written $\boldsymbol{A}^p$ in [4]). Similarly $\boldsymbol{O}^p$ represents the ideal defined in [4, 7.12] and $\mathscr{O}^p$ the corresponding z-filter.