PACIFIC JOURNAL OF MATHEMATICS
Vol. 51, No. 1, 1974

THE MULTIPLIER ALGEBRA OF A CONVOLUTION MEASURE ALGEBRA

KARI YLINEN

In this paper the structure theory of convolution measure algebras due to J. L. Taylor is used in studying the multiplier algebra $M(A)$ of a commutative semi-simple convolution measure algebra A. A criterion is given for the embeddability of $M(A)$ in the measure algebra $M(S)$ on the structure semigroup S of A, and the relationship between the structure semigroups of A and $M(A)$ is investigated in case $M(A)$ is also a convolution measure algebra and S has an identity.

1. Introduction. A convolution measure algebra A is a complex L-space with a multiplication which gives A the structure of a Banach algebra and satisfies certain additional requirements. For precise definitions and the basic theory of convolution measure algebras we refer to J. L. Taylor's paper [11]. A central role in Taylor's theory is played by the structure semigroup S of a commutative convolution measure algebra A. The maximal regular ideal space of A may be identified with the set of semicharacters of the compact commutative topological semigroup S, and some properties of A are reflected in those of S.

For any (complex) commutative Banach algebra A, let $\Delta(A)$ denote the spectrum of A, that is, the space of nonzero multiplicative linear functionals on A, equipped as usual with the relative weak* topology. If A is in addition semisimple, then we denote by A^m the space of all complex-valued functions on $\Delta(A)$ that keep the space $\hat{A}$ of the Gelfand transforms $\hat{x}$ of the elements x of A invariant by pointwise multiplication, i.e., $A^m = \{f: \Delta(A) \rightarrow C \mid f\hat{x} \in \hat{A} \text{ for all } x \in A\}$. It can be easily shown that each $f \in A^m$ determines a unique bounded linear operator $T_f: A \rightarrow A$ satisfying $\widehat{T_f x} = f\hat{x}$, $x \in A$. Then $M(A) = \{T_f \mid f \in A^m\}$ is a Banach algebra under the uniform operator norm, called the *multiplier algebra* of A. For the general theory of multiplier algebras one may consult e.g. Larsen's book [5].

In this paper we study the multiplier algebra of a commutative semi-simple convolution measure algebra A. J. L. Taylor has shown in [11] that A may be naturally embedded in the convolution algebra $M(S)$ of finite regular Borel measures on the structure semigroup S. In § 3 we show that $M(A)$ can be isometrically realized as a subalgebra of $M(S)$ containing the image of A if and only if S has an identity. As is to be expected, the measures then corresponding to isometric onto multipliers have one point support in S. Section 4 gives con-