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ON THE STRUCTURE OF FINITE RINGS II

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In this paper we develop a structure theory for modules and bimodules over complete matrix rings over Galois rings, and we use this module theory to study the additive structure of the components of a Peirce decomposition of a general finite ring.

We recall that any finite ring is the direct sum of rings of prime power characteristic. This follows from noticing that when one decomposes the additive group of a finite ring into its primary components, the components are ideals of prime power characteristic (cf. [4]). We thus restrict ourselves to considering rings of prime power characteristic without loss of generality up to direct sum formation.

We next recall the definition of a Galois ring. Let k, r be positive integers and p be a prime integer. The *Galois ring of characteristic* p^k *and order* p^{kr} is defined to be $Z[x]/(p^k, f(x))$ [8], [10] where Z denotes the rational integers and $f(x) \in Z[x]$ is monic of degree r and irreducible. A Galois ring is uniquely determined up to isomorphism by the integers p, k, and r, and we shall denote the Galois ring of characteristic p^k and order p^{kr} by $G(k, r)$. The prime p will generally be clear from context. Note that $G(1, r) \cong GF(p^r)$ and $G(k, 1) \cong Z/(p^k)$.

If R is a finite ring of characteristic p^k which contains a 1 then R contains a Galois ring $G(k, r)$ for some r which contains the 1 of R. Indeed $Z/(p^k)\cdot 1$ will always be such a ring. Therefore, any finite ring of characteristic p^k is thus a faithful left and right $G(k, r)$-module for some r.

We now seek to develop a module theory for matrix rings over Galois rings. In a sense, the theory is already developed in that a matrix ring over a Galois ring is Morita equivalent to a Galois ring and hence the categories of modules will be category isomorphic, and a module and bimodule theory already is known for modules over Galois rings [11]. However, we seek slightly more information than is given by the category isomorphism from Morita theory. In what follows Q will denote the matrix ring $M_n(G(k, r))$.

PROPOSITION 1. *Let M be a finitely generated left Q-module. Then M is a direct sum of cyclic left Q-modules.*

Proof. Every finitely generated left $G(k, r)$-module is a direct sum of cyclic left $G(k, r)$-modules by Corollary 2 to Proposition 1.1 of [11].