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DETERMINING KNOT TYPES FROM DIAGRAMS OF KNOTS

JOHN R. MARTIN

The word of knot and the characteristics of its double points, both of which may be read from the diagram of knot, are used to give necessary and sufficient conditions for two (oriented) knots to belong to the same (oriented) knot type.

1. Introduction. A knot is a circle imbedded as a polygon in 3-dimensional space R^3. Two knots K and L are called *equivalent* if there is an autohomeomorphism h of R^3 such that $h(K) = L$. Each equivalence class is called a *knot type*. For oriented knots a stronger equivalence may be defined: Two oriented knots K and L are called *O-equivalent* if there is an orientation preserving autohomeomorphism h of R^3 such that h maps K onto L so their orientations match. In this case each equivalence class is called an *oriented knot type*.

In [8] D. E. Penney uses the diagram of a knot to define a "word" for the knot and obtain sufficient conditions for two knots to belong to the same knot type. Penney's results have been generalized by L. B. Treybig in [12] where the concept of the "boundary collection" of a knot is used to give necessary conditions for two knots to belong to the same knot type. The purpose of this paper is to use the word of a knot and the characteristics of its double points, both of which may be read from the diagram of a knot, to give necessary and sufficient conditions for two (oriented) knots to belong to the same (oriented) knot type.

The preliminaries needed for our main results are given in §2. In §3 a relationship between the word of a knot and the characteristics of its double points is derived. This relationship is used to obtain Theorem 3.4 which yields sufficient conditions for two oriented knots to belong to the same oriented knot type. In §4 the following two principal results are obtained: Theorem 4.3 states that two prime knots K and L are equivalent iff there exists a certain finite sequence of words relating a word of K to a word of L. Theorem 4.4 states that two oriented knots K and L are O-equivalent iff there exists a certain finite sequence of words and characteristics relating the word and characteristic of K to those of L.

We remark that in each of Theorems 4.3 and 4.4 the sufficiency part follows from the results developed in §3 while the necessity is an immediate consequence of the classical work of Alexander and Briggs [1].

In §5 the group of a prime word is defined and this is used in