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POWER INVARIANT RINGS

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A ring A is called power invariant if whenever B is a ring such that the formal power series rings $A[[X]]$ and $B[[X]]$ are isomorphic, then A and B are isomorphic. A ring A is said to be strongly power invariant if whenever B is a ring and ϕ is an isomorphism of $A[[X]]$ onto $B[[X]]$, then there exists a B-automorphism ψ of $B[[X]]$ such that $\psi(X) = \phi(X)$. Strongly power invariant rings are power invariant. For any commutative ring A, $A/J(A)^n$ is strongly power invariant, where $J(A)$ is the Jacobson radical of A, and n is any positive integer. A left or right Artinian ring is strongly power invariant. If A is a left or right Noetherian ring, then $A[t]$, the polynomial ring in an indeterminate t over A, is strongly power invariant.

Introduction. Coleman and Enochs [2] raised the following question: Can there be nonisomorphic rings A and B whose polynomial rings $A[X]$ and $B[X]$ are isomorphic? Recently Hochster [4] answered this question in the affirmative. The analogous question about a commutative formal power series ring was raised by O'Malley [7]: If $A[[X]] \cong B[[X]]$, must $A \cong B$? We know no counterexamples.

In this paper all rings are assumed to have identity elements. The Jacobson radical and the prime radical (the intersection of all prime ideals) of a ring A will be denoted by $J(A)$ and $\operatorname{rad}(A)$, respectively. Let $A[[X]]$ be the formal power series ring in a commutative indeterminate X over a ring A, and let β be a central element of $A[[X]]$. Then (β^n) will denote the ideal of $A[[X]]$ generated by β^n for a nonnegative integer n, and $(A[[X]], (\beta))$ denotes the topological ring $A[[X]]$ with the (β)-adic topology. It is well known that $(A[[X]], (\beta))$ is Hausdorff if and only if $\bigcap_{n=1}^{\infty} (\beta^n) = (0)$. The (β)-adic topology is metrizable in the obvious way, and we say that $(A[[X]], (\beta))$ is complete if each Cauchy sequence of $A[[X]]$ converges in $A[[X]]$. Then clearly $(A[[X]], (X))$ is a complete Hausdorff space.

Extending the terminology used in [2], O'Malley [7] defined "power invariant ring" and "strongly power invariant ring" as follows: A ring A is power invariant if whenever B is a ring such that $A[[X]] \cong B[[X]]$, then $A \cong B$. A ring A is said to be strongly power invariant if whenever B is a ring and ϕ is an isomorphism of $A[[X]]$ onto $B[[X]]$, then there exists a B-automorphism ψ of $B[[X]]$ such that $\psi(X) = \phi(X)$.

Let A be a strongly power invariant ring and let ϕ be an isomorphism of $A[[X]]$ onto $B[[X]]$. Then there exists a B-automorphism