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STRICTLY LOCAL SOLUTIONS OF DIOPHANTINE EQUATIONS

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For any system f of Diophantine equations, there exist positive integers $C(f)$, $D(f)$ with the following properties: For any nonnegative integer n, for any prime p, if v is the p-adic valuation, and if a vector x of integers satisfies the inequality

$$v(f(x)) > C(f)n + v(D(f))$$

then there is an algebraic p-adic integral solution y to the system f such that

$$v(x - y) > n .$$

This theorem is proved by techniques of algebraic geometry in the more general setting of Noetherian domains of characteristic zero. When f is just a single equation, the method of Birch and McCann gives an effective determination of $C(f)$ and $D(f)$.

Let R be a Noetherian integral domain, K its field of fractions. We will consider *Henselian discrete valuation rings* R_v (see [4]) containing R, where v is the valuation normalized so that $v(R_v)$ is the set of nonnegative integers (plus ∞). If $f = (f_1, \cdots, f_r)$ is a system of r polynomials in s variables with coefficients in R, and x is an s-tuple with coordinates in an extension ring of R, we set $f(x) = (f_1(x), \cdots, f_r(x))$. We define the valuation of an r-tuple (or s-tuple) to be the minimum of the valuations of its components.

THEOREM. *Assume R has characteristic zero. For each system f of polynomials with coefficients in R, there exists an integer $C(f) \geqq 1$ and an element $D(f) \neq 0$ in R with the following property: For any Henselian discrete valuation ring R_v containing R, and any nonnegative integer n, if an s-tuple x with components in R satisfies the inequality*

$$v(f(x)) > C(f)n + v(D(f)) \tag{1}$$

then there is a zero y of f in R_v such that

$$v(x - y) > n .$$

In particular, if R is the ring of algebraic integers in a number field, and we take $n = 0$, $S =$ set of primes dividing $D(f)$, then we recover Greenleaf's theorem [3] to the effect that if $\mathfrak{p} \notin S$, then every