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A GALOIS THEORY FOR LINEAR TOPOLOGICAL RINGS

B. L. ELKINS

Separable algebras have been studied recently by M. Auslander, D. Buchsbaum and Chase-Harrison-Rosenberg. The question of a Galois theory for linear topological rings opposite to the Krull type theory obtained in the above works was raised by H. Röhrl. In this paper, a Galois theory relating the complete subalgebras of restricted type of a complete algebra A to a set of subgroups of a discrete group G of automorphisms of A is developed.

The notion of a linear topological module has been discussed in [1], [5], [6], [7]; while the concepts pertaining to separables algebras are now available in the monograph [4] for the most part. We employ two results of [3] which we will state below. All rings considered will be commutative with 1.

DEFINITION 0.1 [3]. Two ring morphisms $A \underset{g}{\overset{f}{\rightrightarrows}} B$ are *strongly distinct* if, for each nonzero idempotent $e \in B$, there is $a \in A$ with $f(a)e \neq g(a)e$. Where B is connected, f and g are strongly distinct if and only if they are distinct.

THEOREM 0.2 [3]. *Let G be a finite group of automorphisms of the ring A having (pointwise) fixed ring k. The following statements are equivalent*:

(0) *A is a separable k-algebra* [*and the elements of G are pairwise strongly distinct*].

(1) *There are families of elements of A, $(x_i)_{i=1}^n$, $(y_i)_{i=1}^n$ with*

$$\sum_{i=1}^{n} x_i \sigma(y_i) = \delta_{1\sigma}$$

for each $\sigma \in G$, where $\delta_{1\sigma}$ is the Kronecker delta.

(2) *For each $\sigma \in G \backslash \{1\}$ and each maximal ideal $m < A$, there is $a \in A$ with $a - \sigma(a) \notin m$.*

(3) *For each connected k-algebra B and each pair $A \underset{g}{\overset{f}{\rightrightarrows}} B$ of k-algebra morphism, there is a unique $\sigma \in G$ with $\sigma g = f$.*

Proof. $(0) \rightarrow (1) \rightarrow (2) \rightarrow (0)$ is contained in [3], Theorem (1.3), and the implication $(2) \rightarrow (3)$ is Corollary (3.2) of [3]. We establish $(3) \rightarrow (2)$. Let $m < A$ be a maximal ideal and suppose $\sigma \in G \backslash \{1\}$. Then the