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PROJECTIONS OF UNIQUENESS FOR $L^p(G)$

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Let G be a locally compact, noncompact, unimodular group. For $x \in G$, we denote by L_x, the left translation operator defined on $L^2(G)$ by $L_x f(y) = f(x^{-1}y)$. We let $\mathscr{L}_2(G)$ be the closure, in the weak operator topology, of the algebra generated by the operators $\{L_x: x \in G\}$. For $f \in L^p(G)$, $1 \leqq p \leqq 2$, we let L_f be the closed operator in $L^2(G)$, defined by $L_f g = f * g$, for $g \in L^1(G) \cap L^2(G)$. We prove, under a natural hypothesis on G, that for every $1 < p < 2$, there exists a projection $P \in \mathscr{L}_2(G)$, $P \neq 0$, with the property that if $f \in L^p(G)$, and $PL_f = L_f$, then $f = 0$. Thus P is a projection of uniqueness in the sense that the only element $f \in L^p$, such that the range of L_f is contained in the range of P is the zero element. Another way to express this result is the following: There exists a nontrivial closed subspace of $L^2(G)$, invariant under right translations and which contains no nonzero element of $L^p(G)$.

The additional hypothesis required on G is the following, which will be tacitly assumed in the sequel:

(H) $\mathscr{L}_2(G)$ *is not purely atomic, that is, it is not generated, as a von Neumann algebra, by its minimal projections.*

Hypothesis (H) is certainly satisfied if G is discrete. However, there exist noncompact groups for which (H) is not true. A discussion of hypothesis (H) will be given at the end of this paper.

The progenitor of the present work is a note by Y. Katznelson [11] in which it is proved that there exist nonnegligible subsets of the torus $\boldsymbol{T}$ which are sets of uniqueness for l^p when $p < 2$. Our result coincides with Katznelson's theorem if G is taken to be the group of integers $\boldsymbol{Z}$, with the discrete topology.

A. Figà-Talamanca and G. I. Gaudry have extended Katznelson's result to the case in which G is a locally compact noncompact Abelian group [6]. In fact for the theorem proved in the present paper, we use the techniques of [6] in the framework of I. E. Segal's noncommutative integration theory [18] as applied by R. Kunze [13] and W. Stinespring [19] to the canonical gage space of a locally compact unimodular group.

It is evident that hypothesis (H) plays the role of the hypothesis of nondiscreteness of the character group of G made in [6]. Whereas it is clear that discrete (Abelian) groups cannot have sets of unique-