PACIFIC JOURNAL OF MATHEMATICS
Vol. 51, No. 1, 1974

π-HOMOGENEITY AND π'-CLOSURE OF FINITE GROUPS

ZVI ARAD

The purpose of this paper is to present a proof, under additional conditions, of the following conjecture: Let π be a set of primes, and let all π-subgroups of G be 2-closed. (If $2 \notin \pi$, this condition is satisfied.) If G is π-homogeneous, then G is π'-closed.

All groups considered here are *finite*. If π is a set of prime numbers, we say that the element x of a group G is a *π-element* if $|x|$ is divisible only by primes in π. In particular, one may speak of a *p-element*, p a prime. Similarly, a group G is called a *π-group* if $|G|$ is divisible only by primes in π. In addition, $\pi(G)$ will denote the set of primes dividing $|G|$. The set of primes not in π will be denoted by π'. A group G is termed *π-closed*, if the subset of G consisting of π-elements is a subgroup of G. We say that a group G is *π-homogeneous* if $N_G(H)/C_G(H)$ is a π-group for every nonidentity π-subgroup H of G.

It is well known that π'-closed groups are π-homogeneous. The converse, in general, does not hold. For instance, A_5 is not 5-closed, but it is $5'$-homogeneous.

For $\pi = \{p\}$, p a prime, the conjecture reduces to Frobenius' theorem ([11], Theorem 7.4.5).

The conjecture is closely connected to other well known problems in group theory. The proof of the conjecture would imply the solution of Baer's problem [3] (see also [5], p. 117), the answer to which is not known.

Baer's Problem. Let $\pi \subsetneqq \pi(G)$. Suppose that G is π and π'-homogeneous. Is G a direct product of a π-group and a π'-group?

In order to show the connection with Frobenius' problem, we need some additional notation. For any prime p, we denote by $|G|_p$ the highest power of the prime p that divides $|G|$. Define G to be *weakly π-closed* if for every subgroup U of G the number of π-elements of U is exactly $\prod_{p \in \pi} |U|_p$.

Baer proved that if G is weakly π-closed then G is π'-homogeneous ([2], Lemma 2). Therefore, in the case that $2 \in \pi$, the proof of the above conjecture would imply also a solution of Frobenius' problem ([2], p. 325).