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ENTROPY OF SELF-HOMEOMORPHISMS OF STATISTICAL PSEUDO-METRIC SPACES

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A pseudo-Menger space is a set X together with a function $\theta: X \times X \to \mathscr{D}$, the set of distribution functions, satisfying certain natural axioms similar to those of a pseudo-metric space. Let $T: X \to X$ be a bijection and let θ_T denote the topology generated by $\{T^i U(p, \varepsilon, \lambda): i \in Z, p \in X, \varepsilon > 0, \lambda > 0\}$ where $U(p, \varepsilon, \lambda) = \{q: \theta(p, q)(\varepsilon) > 1 - \lambda\}$. Assume that θ_T is compact. Let $h(T, \theta)$ denote the topological entropy of T with respect to the θ_T topology. The purpose of this note is to show that if one is given a sequence $\{\theta_n\}$ of pseudo-Menger structures on X satisfying $\theta_n(p, q) \geqq \theta(p, q)$ and $\theta_n(p, q) \to \theta(p, q)$ in distribution for all $p, q \in X$ then $h(T, \theta_n) \to h(T, \theta)$. A counterexample is then given to show that, in general, the condition $\theta_n(p, q) \geqq \theta(p, q)$ cannot be removed.

1. The investigation of statistical metric spaces was undertaken by Karl Menger [5] in 1942. Essentially these are spaces in which the "distance" between any two points is given by a probability distribution function. Our purpose is to investigate the behavior of the topological entropy of a self-homeomorphism of a compact Menger space under perturbations of these distribution functions. We proceed to give precise definitions.

2. Preliminaries. Let I denote the closed unit interval, Q^+ the positive rationals, Z^+ the positive integers, and $\mathscr{D}$ the set of all left-continuous monotone increasing functions $F: \boldsymbol{R} \to I$ satisfying $F(0) = 0$ and $\sup F(x) = 1$. Let H be the function defined by: $H(t) = 0$ for $t \leqq 0$ and $H(t) = 1$ for $t > 0$.

Throughout our discussion, X will be a fixed set. Let $\mathscr{F}$ denote the collection of all functions $\theta: X \times X \to \mathscr{D}$. For convenience we shall often write θ_{pq} in place of $\theta(p, q)$. A *statistical pseudo-metric space* is an ordered pair (X, θ) where $\theta \in \mathscr{F}$ satisfies

(a) $\theta_{pq} = \theta_{qp}$ for all $p, q \in X$.

(b) $\theta_{pq}(a + b) = 1$ whenever $\theta_{pr}(a) = \theta_{rq}(b) = 1$.

(c) $\theta_{pp} = H$ for all $p \in X$.

If, in addition, θ satisfies

(d) $\theta_{pq} = H$ only if $p = q$

then (X, θ) is a *statistical metric space*.

Let $\mathscr{S}$ denote the collection of all θ for which (X, θ) is a statistical pseudo-metric space.

A *triangular norm* is a function $\Delta: I \times I \to I$ which is associative,