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SEPARATE CONTINUITY AND JOINT CONTINUITY

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The main theorem is somewhat stronger than the following statement: Let X be either a locally compact Hausdorff space of a complete metric space, let Y be a compact Hausdorff space and let Z be a metric space. If a map $f: X \times Y \to Z$ is separately continuous, then there is a dense G_δ-set A in X such that f is jointly continuous at each point of $A \times Y$. This theorem has consequences such as Ellis' theorem on separately continuous actions of locally compact groups on locally compact spaces and the existence of denting points on weakly compact convex subsets of locally convex metrizable linear topological spaces.

0. Introduction. The consideration of separate continuity *vis-à-vis* joint continuity goes back, at least, to Baire (1899), whose work is the prototype of all the subsequent investigations on this subject by many mathematicians. The general problem is (P): find conditions on topological spaces X, Y, and Z so that each separately continuous function $f: X \times Y \to Z$ (i.e., function continuous in each variable while the other variable is fixed) is jointly continuous at points of a "substantial" (in some topological sense) subset of $X \times Y$. As far as we know, all the available answers to this question require that either X or Y be metrizable or satisfy some sort of countability condition (e.g. [4, § 5 Problem 23] and [9, Theorem 3]). This requirement severely restricts their applications. For instance, the proof of Ellis' theorem [7, Theorem 1] is fairly complicated because a step by step reduction of the general case to the metrizable case is involved. This also explains why the theorems on weak-compact sets in [18] require the separability condition.

However, the recent result of Troyanski [21] on renorming a Banach space that is generated by a weak-compact set enables one to drop the separability condition from many theorems on weak-compact subsets of Banach spaces. Indeed, it is relatively simple to deduce the following answer to problem (P) from Troyanski's renorming theorem: Let X and Y be compact Hausdorff spaces and let f be a *bounded* separately continuous real-valued function on $X \times Y$. Then there are dense G_δ-sets A and B of X and Y respectively such that f is jointly continuous at each point of $A \times Y \cup X \times B$. This indicates that some good answers to problem (P) have been overlooked in the past.