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REGULAR COMPLETIONS OF CAUCHY SPACES

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A uniform convergence space is a generalization of a uniform space. The set of all Cauchy filters of some uniform convergence space is called a Cauchy structure. We give necessary and sufficient conditions for the Cauchy structure of some totally bounded uniform convergence space to be precompact; i.e., have a regular completion. Also, it is shown that there is an isomorphism between the set of ordered equivalence classes of strict regular compactifications of a completely regular convergence space and the set of ordered precompact Cauchy structures inducing the given convergence structure.

Preliminaries. Kowalsky [5] has studied completions using only Cauchy filters, described axiomatically, and not necessarily those of a uniform convergence space. This has led others to the notion of a Cauchy space, which is described axiomatically in [2]. The reader is referred to [6], [7], and [8] for a discussion of completions of Cauchy spaces.

For basic definitions of convergence spaces and uniform convergence space, see [3] and [1]. A Hausdorff convergence space (S, q) is compatible with a uniform convergence space iff it satisfies the "Limitierungsaxiom": $\mathfrak{F} \cap \mathfrak{G}$ q-converges to x whenever $\mathfrak{F}$ and $\mathfrak{G}$ both q-converge to x. *We will make the assumption that all convergence spaces in this paper satisfy this axiom.* The closure operator in a convergence space (S, q) will be denoted by Γ_q. A Hausdorff convergence space (S, q) is called *regular* if it has the property that $\Gamma_q\mathfrak{F}$ (the filter generated by $\{\Gamma_q F \mid F \in \mathfrak{F}\}$) q-converges to x whenever $\mathfrak{F}$ q-converges to x. The filter $\dot{x}$ denotes the set of all subsets of S containing the set $\{x\}$. If filters $\mathfrak{F}$ and $\mathfrak{G}$ contain disjoint sets, we write "$\mathfrak{F} \vee \mathfrak{G} = 0$". The term "ultrafilter" will be abbreviated "u.f."; uniform convergence space will be abbreviated "u.c.s.".

A *Cauchy structure* $\mathscr{C}$ on a set S is characterized axiomatically in [2] as follows: (1) $\dot{x} \in \mathscr{C}$ for each $x \in S$; (2) $\mathfrak{F} \in \mathscr{C}$ and $\mathfrak{G}$ finer than $\mathfrak{F}$ implies $\mathfrak{G} \in \mathscr{C}$; (3) $\mathfrak{F}, \mathfrak{G} \in \mathscr{C}$ and $\mathfrak{F} \vee \mathfrak{G} \neq 0$ implies $\mathfrak{F} \cap \mathfrak{G} \in \mathscr{C}$. The pair $(S, \mathscr{C})$ is called a *Cauchy space.* It should be pointed out that the Cauchy space axioms of [2] are stricter than those of [5] and [7].

A Cauchy space $(S, \mathscr{C})$ induces a convergence structure q in the following way: $\mathfrak{F}$ q-converges to x iff $\mathfrak{F} \cap \dot{x} \in \mathscr{C}$. Conversely, if (S, q) is a Hausdorff convergence space, then define the *associated Cauchy structure* $\mathscr{C}$ on S: $\mathfrak{F} \in \mathscr{C}$ iff $\mathfrak{F}$ q-converges. Note that $(S, \mathscr{C})$ induces