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EXTENDED PRIME SPOTS AND QUADRATIC FORMS

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Some of the local theory of extended prime spots on fields is developed here, with two applications in mind. In the first, two analogues to the Hasse-Minkowski theorem on equivalence of quadratic forms over global fields are developed, based on the notion of an ultracompletion of a field at an extended prime spot. They deal, respectively, with equivalence of quadratic forms over a simple transcendental extension of a global field, and with the reduced Witt ring of a general field. Examples illustrate problems involving the further extension of the global theory of quadratic forms. In the second application Harrison and Warner's ultracompletions of a field at a finite or infinite prime are shown to be essentially ultracompletions at associated extended prime spots.

Extended prime spots were first introduced to provide a setting for a generalization of the weak approximation theorem for independent absolute values. In § 1, we recall from [3] the definitions of extended absolute values and extended prime spots and we define and study ultracompletions of fields at extended absolute values. An alternative approach to ultracompletions of fields at Harrison primes [7] is sketched in § 2. The analogues to the Hasse-Minkowski theorem are discussed in § 3; this section owes a great debt to work of Milnor [14] and Pfister [15]. Finally, in an appendix (§ 4) we sketch a theory of Henselizations of extended absolute values. The last three sections of this paper are essentially independent of each other.

Throughout this paper F will denote a field. $F^{\times}$ denotes its multiplicative group of nonzero elements. Z, Q, R, and C denote the sets of integers, rational numbers, real numbers and complex numbers, respectively. $A\backslash B$ denotes the set of elements of the set A which are not in the set B.

1. Ultracompletions. We recall some concepts from [3, especially § 5]. An extended absolute value on F is a map $\varphi: F \rightarrow R \cup \{\infty\}$ with $\varphi(a+b) \leqq \varphi(a) + \varphi(b)$, $\varphi(ab) = \varphi(a)\varphi(b)$ (when defined), $\varphi(a) \geqq 0$, $\varphi(1) = 1$ and $\varphi(0) = 0$ for all $a, b \in F$. (We do not define $0 \cdot \infty$ or $\infty \cdot 0$.) The extended absolute values on F are precisely those maps obtained by composing a place on F with an absolute value on the residue class field of the place. (We intend that the composite function map to ∞ those elements of F which the place maps to ∞.) For, if φ is an extended absolute value on F, then $\varphi^{-1}(R)$ is a valua-