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ON DEFINING A SPACE BY A WEAK BASE

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Arhangel'skii has defined the concepts of a weak base and a g-first countable space ($= gf$-axiom of countability $=$ weak first axiom of countability). Here a g-second countable space and a g-metrizable space are defined and discussed, in particular in relation to metrizability.

We wish to discuss a means of defining topological spaces which may deserve to be better known. We begin with a slight modification of a definition due to Arhangel'skii [4, p. 129].

DEFINITION 1.1. For a topological space X and a point x of X, a collection T_x of subsets of X is called a collection of *weak neighborhoods* of x if each member of T_x contains x, for any two members of T_x their intersection is also a member of T_x, and the following is true: Letting $\mathscr{B} = \bigcup \{T_x \mid x \in X\}$, $\mathscr{B}$ is a *weak base* for X if a set U is open in X if and only if for every point x in U there exists a $B \in T_x$ such that $B \subset U$.

1.2. It is clear that any set X may be topologized by this natural method. To illustrate, we consider three known examples of spaces in which the topology is defined in this manner. Let X be the plane, and for each point x of X, we now define the collection T_x for each of these three examples:

(a) T_x consists of all sets which form a "plus at x", namely, sets which are the union of a horizontal open interval and a vertical open interval each of which contains the point x. (This example of J. Novak [27] may have its topology defined in some interesting alternative manners. See [1, Example 2], [3, p. 30], and [22].)

(b) T_x consists of all sets "radial at x", namely, sets which contain a line segment through x in every direction. (This is an example of Williams in [13].)

(c) $T_x = \{V_n(x) \mid n = 1, 2, \cdots\}$, where $V_n(x)$ is the translation of $V_n(0)$ by x, and $V_n(0)$ is that subset of the open $1/n$ disc with center at the origin which is obtained by removing all points of the open second and fourth quadrants. (This is an example of Meyer in [21, 3.10].)

Example (c) is immediately seen to be a g-first countable space in the sense of the following definition also due to Arhangel'skii [4, p. 129]. The reader may also notice that example (a) is a g-first countable space, while example (b) is not.