PACIFIC JOURNAL OF MATHEMATICS
Vol. 52, No. 1, 1974

FIXED POINT SETS OF POLYHEDRA

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It is shown that every closed nonempty subset of a polyhedron can be the fixed point set of a suitable self-map if the polyhedron satisfies a certain connectedness condition. Hence the same is true for all compact triangulable manifolds with or without boundary. The proof uses existing results on deformations of polyhedra with a minimum number of fixed points if the dimension of the polyhedron is at least two, and on self-maps of dendrites with given fixed point sets if the dimension of the polyhedron is one.

1. The ploblem. The following problem has been investigated by H. Robbins, L. E. Ward, Jr., and the author in recent years: *If X is a topological space and A a closed subset of X, when does there exist a continuous self-mapping of X with A as its fixed point set?* Several cases are known where A need not satisfy any restrictions apart from the obvious one that it must be nonempty if X has the fixed point property.

L. E. Ward, Jr. [7] suggested the term "*complete invariance property*" if a space has the property that each of its nonempty closed subspaces can be the fixed point set of a self-map. Spaces known to have the complete invariance property include the n-cells [2], dendrites [4] and compact manifolds without boundary [6]. L. E. Ward, Jr. [7] extended this list with several other spaces, among them arcwise connected subspaces of locally smooth dendroids and a certain class of Peano continua. He also found an interesting example which shows that a tree need not have the complete invariance property, and hence that a generalization of existing results to nonmetric spaces presents difficulties.

It is the purpose of the present paper to show that all polyhedra which satisfy a certain connectedness condition have the complete invariance property. More precisely, they have to be of type W (as defined in § 2) if their dimension is greater than one (Theorem 3.1), and to be connected if their dimension is one (Theorem 3.2). It follows easily that all compact manifolds with or without boundary possess the complete invariance property (Corollary 3.3). The proof of Theorem 3.1 leans heavily on the construction of deformations of polyhedra with a minimum number of fixed points, which is now easily accessible in the recent book by R. F. Brown [1].

I wish to thank Professor L. E. Ward, Jr. for private communications of some of his results.