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ON THE NIELSEN NUMBER OF A FIBER MAP

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Suppose $\mathscr{T}=\{E, \pi, B, F\}$ is a fiber space such that $0 \rightarrow \pi_1(F) \xrightarrow{i_\#} \pi_1(E) \xrightarrow{\pi_\#} \pi_1(B) \rightarrow 0$ is exact. Suppose also that the above fundamental groups are abelian. If $f: E \rightarrow E$ is a fiber preserving map such that $f_\#(\alpha)=\alpha$ if and only if $\alpha=0$, then it is shown that $R(f)=R(f')\cdot R(f_b)$ where $R(h)$ is the Reidemeister number of the map h.

A product formula for the Nielsen number of a fiber map which holds under certain conditions was introduced by R. Brown. Let $\mathscr{T}=\{E, \pi, L,(p, q), s^1\}$be a principal s^1-bundle over the lens space $L(p, q)$, where $\mathscr{T}$ is determined by $[f_j] \in [L(p, q), cp^\infty] \simeq H^2(L(p, q), z) \simeq z_p$. Let $f: E \rightarrow E$ be a fiber preserving map such that $f_{b\#}(1)=c_2, f'_\#(\bar{l}_p)=\bar{c}_1$, where 1 generates $\pi_1(s^1) \simeq z$ and $\bar{l}_p$ generates $\pi_1(L(p, q)) \simeq z_p$. Then the Nielsen numbers of the maps involved satisfy

$$N(f)=N(f_b)\cdot(d, 1-c_1, s) ,$$

where $d=(j, p)$ and $s=j/p(c_1-c_2)$.

I. Introduction. Let $\mathscr{T}=\{E, \pi, B, F\}$ be a fiber space. Any fiber preserving map $f: E \rightarrow E$ induces maps $f': B \rightarrow B$, and, for each $b \in B$, $f_b: \pi^{-1}(b) \rightarrow \pi^{-1}(b)$, where $\pi^{-1}(b) \simeq F$. The map f will be called a fiber map (or bundle map if $\mathscr{T}$ is a bundle).

Let $N(g)$ denote the Nielsen number of a map g. The Nielsen number, $N(g)$, serves as a lower bound on the number of fixed points of a map homotopic to g, and under certain hypotheses, there exists a map homotopic to g with exactly $N(g)$ fixed points. R. Brown and E. Fadell ([2] and [3]) proved the following:

THEOREM. *Let $\mathscr{T}=\{E, \pi, B, F\}$ be a locally trivial fiber space, where E, B, and F are connected finite polyhedra. Let $f: E \rightarrow E$ be a fiber map. If one of the following conditions holds:*

(i) $\pi_1(B)=\pi_2(B)=0$.

(ii) $\pi_1(F)=0$.

(iii) *$\mathscr{T}$ is trivial and either $\pi_1(B)=0$ or $f=f' \times f_b$ for all $b \in B$ then $N(f)=N(f')\cdot N(f_b)$ for all $b \in B$.*

These strong restrictions on the spaces involved eliminate some interesting fiber spaces. For example, any circle bundle over B with $\pi_1(B) \neq 0$ is excluded. Furthermore, if $\pi_1(B)=\pi_2(B)=0$, then the total space E is $B \times S^1$.

This paper has two objectives. The first is to try to generalize