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EXTREME OPERATORS ON CHOQUET SIMPLEXES

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If K is a Choquet simplex and X is a metrizable compact Hausdorff space, we let $\partial_S K$ denote the set of extreme points of K with the facial topology and let $S(L(C(X), A(K)))$ denote the set of continuous operators from $C(X)$ into $A(K)$ with norm not greater than 1. Our main purpose in this paper is to characterize the extreme points of $S(L(C(X), A(K)))$. We show that T is an extreme point of $S(L(C(X), A(K)))$ if and only if its adjoint T^* sends extreme points of K into $X \cup -X \subseteqq C(X)^*$, also, the set of extreme points of $S(L(C(X), A(K)))$ equals $C(\partial_S K, X \cup -X)$.

1. Suppose E_1, E_2 are two real Banach spaces, we let $S(E_1)$ denote the unit ball of E_1 and let $L(E_1, E_2)$ be the set of continuous linear operators from E_1 into E_2. Following Morris and Phelps [7], we call an operator T in $S(L(E_1, E_2))$ a *nice operator* if its adjoint T^* sends extreme points of $S(E_1^*)$ into extreme points of $S(E_2^*)$. It is clear that if T is a nice operator, then T is an extreme point of $S(L(E_1, E_2))$. The converse is in general not true and there are various literatures dealing with this problem under different hypotheses (c.f. [2], [5], [9]). In [2], Blumenthal, Lindenstrauss, Phelps proved the following: *Suppose $E_1 = C(X)$, $E_2 = C(Y)$ where X, Y are compact Hausdorff spaces with X metrizable, then T is an extreme point of $S(L(C(X), C(Y)))$ if and only if there exists a continuous map $\varphi: Y \to X$ and a continuous function $\lambda \in C(X)$, $|\lambda| = 1$ such that $(Tf)(y) = \lambda(y)f(\varphi(y))$ for $y \in Y$ and $f \in C(X)$.* As a simple consequence, we see that in such case, T is a nice operator. Our main purpose in this paper is to prove a similar characterization for an extreme point $T \in S(L(C(X), A(K)))$ where X is a metrizable compact Hausdorff space and $A(K)$ is the set of continuous affine functions on a Choquet simplex K.

Suppose K is a Choquet simplex, we let ∂K be the set of extreme points of K and let $\partial_S K$ be the set ∂K with the *facial topology*. A necessary and sufficient condition for a real-valued function f on $\partial_S K$ to be continuous is that for any $a \in A(K)$, there exists $b \in A(K)$ such that $b(x) = f(x) \cdot a(x)$, $x \in \partial K$ (c.f. [1]). In §2, we extend this property to an E-valued function where E is a Fréchet space and we will make use of the results in the next two sections. In §3, we prove our main theorem: *let K be a Choquet simplex and X a metrizable compact Hausdorff space, then $T \in S(L(C(X), A(K)))$ is an extreme point if and only if there exists $\varphi \in A(K, C(X)^*)$ (set of affine w^*-continuous functions from K into $C(X)^*$) and a $\lambda \in A(K)$ such that*