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REPRODUCING KERNELS AND OPERATORS WITH A CYCLIC VECTOR I

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In this paper a study is begun of the complete unitary invariant $((1 - wT)^{-1}e, (1 - zT)^{-1}e)$, first considered by Livsic in his paper 'On Spectral Resolution of Linear Nonself Adjoint Operators' Mat. Sb., 34 (76), 1954, 145–199, of a triple (T, H, e) where T is a bounded linear operator on a Hilbert space H and e is a cyclic vector for T in H, as a reproducing kernel. One of the important points is the construction of a subset of the group algebra of the torus closed under pointwise addition and convolution. This obviously will generate a ring called the K-ring. A study of this ring will be done later.

Several other theorems and constructions are also given.

Introduction. Let T be a bounded linear operator on a Hilbert space H with a topologically cyclic vector e in H. In this paper we wish to study certain analytic functions associated with the triple (T, H, e) for the sake of the problem of invariant subspaces of T in H. (See also [8] and [15].)

The paper is divided into six sections. In § 1 we present some facts about reproducing kernels with analyticity properties. In § 2 we consider a triple (T, H, e) of the above type. H can then be represented as a Hilbert space of conjugate analytic functions $\alpha_e[H]$ with a reproducing Kernel K. T^* on H assumes the form of reverse shift on the Taylor coefficients of functions in $\alpha_e[H]$. (See also [11] or [19].) In § 3 we recover (T, H, e) from the reproducing kernel of $\alpha_e[H]$ in two ways. The notion of an analytic function of positive definite type is introduced and it is shown that only these can arise as reproducing kernels of $\alpha_e[H]$. These functions are also related to invariant subspaces (§ 4). In § 5 a category of triples is constructed and it is connected to the harmonic analysis of the two-torus via the analytic functions of positive definite type. Section 6 consists of some examples and counterexamples about the analytic functions of positive definite type.

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1. Reproducing kernels and analytic functions of positive type. We start with the definition of a reproducing kernel. Let