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FUNDAMENTAL GROUPS OF COMPACT COMPLETE LOCALLY AFFINE COMPLEX SURFACES, II

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The present article is a continuation of a recent paper by J. P. Fillmore and the author on properly-acting groups Γ of complex affine motions of C^2 such that $\Gamma\backslash C^2$ is compact. In that paper, it was proved that such a group has a normal subgroup Γ_0 of finite index which is either free abelian of rank four or has generators A, B, C, D, with relations

$$ABA^{-1}B^{-1} = C^k (k \geqq 1)$$

and C and D central.

Here we build on this description up to finite index to determine the groups Γ themselves.

1. Introduction. A complete locally affine complex surface X has an orbit-space representation $X = \Gamma\backslash C^2$, where the fundamental group Γ of X is a properly-acting group of complex affine transformations of C^2. Two such surfaces $\Gamma\backslash C^2$ and $\Gamma'\backslash C^2$ are isomorphic if and only if Γ and Γ' are conjugate subgroups of the group $A(2, C)$ of all complex affine motions of C^2. Elements of $A(2, C)$ are taken as nonsingular complex matrices $\begin{pmatrix} a & b & r \\ c & d & s \\ 0 & 0 & 1 \end{pmatrix}$ and elements of C^2 are taken as column vectors $\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$; then $A(2, C)$ acts on the left on C^2 in the usual way.

Fillmore and Scheuneman [2] have shown the following:

THEOREM 1.1. *Let $\Gamma\backslash C^2$ be a compact complete locally affine complex surface. Then:*

(i) *Γ is conjugate in $A(2, C)$ to a subgroup of the group G of all matrices of the form $\begin{pmatrix} 1 & b & r \\ 0 & d & s \\ 0 & 0 & 1 \end{pmatrix}$, and hence may be considered a subgroup of G;*

(ii) *The homomorphism $\begin{pmatrix} 1 & b & r \\ 0 & d & s \\ 0 & 0 & 1 \end{pmatrix} \mapsto d$, when restricted to Γ, has a kernel Γ_0 which is either free abelian of rank four, or has generators A, B, C, D with relations $ABA^{-1}B^{-1} = C^k (k \geqq 1)$ and C and D central in Γ_0;*

(iii) *The image of Γ under the above homomorphism is a finite cyclic group of order $t = 1, 2, 3, 4, 5, 6, 8, 10,$ or 12.*