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CATEGORY THEORY APPLIED TO PONTRYAGIN DUALITY

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A proof of the Pontryagin duality theorem for locally compact abelian (LCA) groups is given, using category-theoretical ideas and homological methods. The proof is guided by the structure within the category of LCA groups and does not use any deep results except for the Peter-Weyl theorem. The duality is first established for the subcategory of elementary LCA groups (those isomorphic with $T^i \oplus Z^j \oplus R^k \oplus F$, where T is the circle group, Z the integers, R the real numbers, and F a finite abelian group), and through the study of exact sequences, direct limits and projective limits the duality is expanded to larger subcategories until the full duality theorem is reached.

Introduction. In this note we present a fairly economical proof of the Pontryagin duality theorem for locally compact abelian (LCA) groups, using category-theoretic ideas and homological methods. This theorem was first proved in a series of papers by Pontryagin and van Kampen, culminating in van Kampen's paper [5], with methods due primarily to Pontryagin. In [10, pp. 102-109], Weil introduced the simplifying notion of compactly generated group and explored the functorial nature of the situation by examining adjoint homomorphisms and projective limits. Proofs along the lines of Pontryagin-van Kampen-Weil appear in the books by Pontryagin [7, pp. 235-279] and Hewitt and Ross [2, pp. 376-380]. A different proof based on abstract Fourier analysis was given by Cartan and Godemont [1]; similar methods are also used by Rudin [9, pp. 27-29] and Heyer [3, pp. 148-161]. Negrepontis [6, pp. 239-252] presented the theorem in light of category theory, but for the most part used different methods and more structure theory than is used here.

The proof we present is based on ideas used previously by Hofmann [4, pp. 109-117] and the author [8]. It rests neither on the structure theorem for compactly generated LCA groups (see [2], [6], [7]) nor on the structure of the L^1-algebra of G (see [6] and [9]). The only deep result we require is the classical Peter-Weyl theorem, applied in our case to compact abelian groups, of course.

Definitions and preliminaries. L is the category of LCA groups, with continuous homomorphisms as morphisms. Groups will be written additively. Pointwise addition of functions makes each $\mathrm{Hom}(G, H)$ an abelian group and L an additive category. R denotes the additive real numbers with the usual topology, Z is the subgroup of R con-