PACIFIC JOURNAL OF MATHEMATICS
Vol. 52, No. 2, 1974

ON THE DIVISIBILITY OF KNOT GROUPS

KUNIO MURASUGI

Some conditions for the knot group to be an R-group, i.e., the group in which the extraction of roots is unique, will be discussed in this paper. In particular, the group of a product knot is an R-group iff the knot group of each component is an R-group. For a fibred knot, a sufficient condition for its group to be an R-group will be given.

A group G is called an *R-group* if for every pair of elements x and y, and every natural number n, it follows from $x^n = y^n$ that $x = y$. In other words, G is an R-group if G has not more than one solution for every equation $x^n = a$. If G is an R-group, G is locally infinite. The converse, however, need not be true even if G is restricted to the group of a knot in S^3. For example, let G be the group of $K(m, n)$, the torus knot of type (m, n). G has a presentation $G = (a, b: a^m = b^n)$. Then the equation $x^m = a^m$ has infinitely many distinct solutions, $x = a, (ba)a(ba)^{-1}, (ba)^2a(ba)^{-2}, \cdots$.

This observation gives immediately a negative answer to Problem N in [3]. Neuwirth asks if a knot group can be ordered. In fact, the group of $K(m, n)(|m|, |n| \geqq 2)$ cannot be ordered, since an ordered group is always an R-group. Therefore, Problem N now leads slightly weaker problems: Can a knot group other than torus knot groups be ordered? Or, is a knot group other than torus knot groups an R-group?

The purpose of this paper is to give a sufficient condition for the group of a fibred knot to be an R-group. (See Theorem 2.) Using this condition, we can prove, for example, that the group of the figure eight knot is an R-group. (See Proposition 3 or Proposition 5.)

1. Statement of main results. To make our statement precise, we will introduce some concepts relevant to an R-group.

DEFINITION 1. Let $n > 1$ be an integer. A group G is said to be n-divisible if for any pair of elements x and y in G it follows from $x^n = y^n$ that $x = y$.

Therefore, a group G is an R-group if G is n-divisible for every n. However, n may be restricted to a prime number. In fact, we have the following easy

PROPOSITION 1. *G is mn-divisible iff G is m- and n-divisible.*