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PROJECTIVE PSEUDO COMPLEMENTED SEMILATTICES

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This paper is concerned with the properties of free, and projective pseudo complemented semilattices (PCSL).

It is proved that a projective PCSL is complemented and all its chains and disjointed subsets are countable, and that a Boolean algebra is projective in the category of PCSL if and only if it is projective in the category of Boolean algebras. Further, necessary and sufficient conditions are established for a finite PCSL to be projective.

1. Preliminaries. A *semilattice* A is a partially ordered set closed under meets. If A has a least element we will denote it by 0. We say that a^* is the *pseudo complement* of $a \in A$, A a semilattice with 0, if we have (i) $a \cdot a^* = 0$, (ii) If $ab = 0$ then $b \leqq a^*$, for $b \in A$. Clearly pseudo complements are unique when they exist. A semilattice with 0 called a *pseudocomplemented semilattice* (PCSL) if each element has a pseudo-complement. A PCSL has a greatest element, 0^*, which we denote by 1. A function $f: A \rightarrow B$, A, B PCSL's, is called a homomorphism if $f(ab) = f(a) \cdot f(b)$, $f(a^*) = f(a)^*$ for $a, b \in A$. We observe that $f(0) = 0$, and $f(1) = 1$. For $S \subseteqq A$ let $S^* = \{x^*: x \in S\}$.

It is easily shown that the following identities are true in any PCSL.

(1)	$xy = yx$	(13)	$(xy)^* = (x^{**}y^{**})^*$
(2)	$x(yz) = (xy)z$	(14)	$x^*y^{**} = 0 \leftrightarrow x^*y^* = x^*$
(3)	$xx = x$	(15)	$xy = 0 \leftrightarrow x \leqq y^*$
(4)	$0 \cdot x = 0$	(16)	$x(xy)^* = xy^*$
(5)	$x(xy)^* = xy^*$	(17)	$x(x^*y)^* = x$
(6)	$x0^* = x$	(18)	$x^*(xy)^* = x^*$
(7)	$0^{**} = 0$	(19)	$x^*(x^*y)^* = x^*y^*$
(8)	$x \leqq x^{**}$	(20)	$x^{**}(x^*y)^* = x^{**}$
(9)	$x \leqq y \rightarrow y^* \leqq x^*$	(21)	$x^{**}(xy)^* = x^{**}y^*$
(10)	$x \leqq y \rightarrow x^{**} \leqq y^{**}$	(22)	$(xy)^*(xy^*)^* = x^*$
(11)	$x^{***} = x^*$	(23)	$(xy)^{**} = x^{**}y^{**}$
(12)	$x^*y^* = (x^*y^*)^{**}$		

The definitions of the concepts discussed in this paper may be found in References 3, 4, 5, and 7.

2. Free PCSL.

LEMMA 2.1. *Let X freely generate the* PCSL F. *Then*