ABSOLUTE CONTINUITY FOR ABSTRACT WIENER SPACES

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Certain smooth homeomorphisms on an abstract Wiener space are shown to induce a measure that is absolutely continuous with respect to an abstract Wiener measure which is the measure determined on a Banach space from the canonical normal distribution on a Hilbert space by the completion of the Hilbert space with respect to a measurable seminorm. The notion of an abstract Wiener space is an abstraction of one technique to show the countable additivity for Wiener measure, the measure for Brownian motion. The generalizations of absolute continuity obtained here reduce exactly to the well known results for absolute continuity for Gaussian measures.

Specifically, it is shown that if T is a C^1 diffeomorphism of the Banach space such that $T = I + \psi$ where I is the identity and ψ takes values in the Hilbert space associated with the abstract Wiener space and is differentiable, then the measure induced on the Banach space by the transformation, T, is absolutely continuous with respect to the abstract Wiener measure.

2. Preliminaries. Let (i, H, B) be an abstract Wiener space where H is a separable Hilbert space, B is a separable Banach space obtained from H by completing H with respect to the measurable seminorm, $|\cdot|$, and $i: H \rightarrow B$ is the continuous canonical injection. \mathscr{T} will denote the Borel σ -algebra on B. For fundamental notions about these measures the reader is referred to [1, 3].

Let ν be the canonical normal distribution on H with variance parameter t = 1 and let μ be the measure induced on B as the projective limit of the family of cylinder set measures on B obtained from ν and the continuous canonical injection, i. Assuming the variance parameter is 1 is inessential for the results for absolute continuity; any variance parameter $t \in (0, \infty)$ could be used with trivial changes in the subsequent results for absolute continuity. For some previous related results on absolute continuity, the reader is referred to [5].

The following definition provides some notation that is used subsequently.

DEFINITION. $\mathscr{P} = \{P: P \text{ is a finite dimensional projection on } H$ with $PH \subset jB^*\}$ where B^* is the topological dual of B and $j = i^*$.