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COMPARISON OF DE RHAM AND DOLBEAULT COHOMOLOGY FOR PROPER SURJECTIVE MAPPINGS

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In this paper it is shown that if $\pi: \tilde{X} \to X$ is a proper holomorphic surjection of equidimensional complex manifolds then the induced mapping $\pi^*: H^q(X, \Omega_X^p) \to H^q(\tilde{X}, \Omega_{\tilde{X}}^p)$ on Dolbeault groups is injective. As a consequence one obtains the inequality $h^{p,q}(X) \leqq h^{p,q}(\tilde{X})$ for the Hodge numbers of X and $\tilde{X}$. This result is valid also in the case of vector bundle coefficients, and can be generalized to the case of nondiscrete fibres of the mapping π (non equidimensional case) by the imposition of a Kählerian condition on $\tilde{X}$. Corresponding results for differentiable mappings are formulated and proved. Illustrative examples are provided to show the necessity of the various assumptions made.

1. Introduction. Let $\pi: \tilde{X} \to X$ be a surjective proper holomorphic mapping of complex manifolds[1]. Our main result in this paper (Theorem 4.1) asserts that if $\tilde{X}$ is a Kähler manifold, then the mapping π induces injections

$$(1.1) \qquad \begin{aligned} \pi^*: H^q(X, \Omega_X^p) &\longrightarrow H^q(\tilde{X}, \Omega_{\tilde{X}}^p) \\ \pi^*: H^r(X, \boldsymbol{C}) &\longrightarrow H^r(\tilde{X}, \boldsymbol{C}) \end{aligned}$$

on the Dolbeault and de Rham groups, respectively. A consequence of this is that we have inequalities

$$(1.2) \qquad \begin{aligned} b_j(\tilde{X}) &\geqq b_j(X) \\ h^{p,q}(\tilde{X}) &\geqq h^{p,q}(X) \end{aligned}$$

for the Betti numbers and Hodge numbers respectively (in the case that $\tilde{X}$ and X are compact, for instance). If $\pi: \tilde{Y} \to Y$ is a proper surjective differentiable mapping of even dimensional orientable manifolds and $\tilde{Y}$ is a symplectic manifold, then there is a natural generalization of the notion of the "degree of π". Under the hypothesis that this degree is not zero,

$$(1.3) \qquad \pi^*: H^r(Y, \boldsymbol{R}) \longrightarrow H^r(\tilde{Y}, \boldsymbol{R}) ,$$

is an injection (Theorem 4.4) (cf. also Borel-Haefliger [2]).

In the case that X and $\tilde{X}$ above have the same dimension, then the conclusion (1.1) and (1.2) still holds without any Kähler assumption,

[1] All manifolds considered in this paper are assumed to be paracompact.