PACIFIC JOURNAL OF MATHEMATICS
Vol. 53, No. 1, 1974

ON SYMMETRY OF SOME BANACH ALGEBRAS

HORST LEPTIN

A Banach *-algebra is called symmetric, if the spectra of elements of the form a^*a contain only nonnegative real numbers. Symmetric Banach *-algebras have a series of important properties, especially with respect to their representation theories. Here it is proved that tensoring with finite dimensional matrix rings preserves symmetry. As an application it is shown that the category of locally compact groups with symmetric L^1-algebras is closed under finite extensions.

In recent years there was a growing interest in the problem of symmetry of involutive Banach algebras. In particular very substantial progress has been made towards a solution of the problem of characterizations of such locally compact groups G for which the group algebra $L^1(G)$ is symmetric. The most striking results in this direction are due to J. Jenkins, who first proved that the discrete "$ax + b$"-group has a nonsymmetric algebra [3] and that the same holds for noncompact semisimple Lie groups [4] (independently this was also proved—but not published—by R. Takahashi). On the other hand, Hulanicki proved symmetry for discrete nilpotent groups (Studia Math. 35) and for class finite groups (Pacific J. Math. 18). Moreover, in Studia Math. 48 I obtained the same results for connected nilpotent Lie groups of class 2.

In [1] D. W. Bailey states a theorem (Theorem 2, p. 417) that a semi-direct product extension of a locally compact group G with a finite group F has a symmetric group algebra, if G has. As (implicitely) in [2] and as in the present note this is reduced to the preservation of symmetry under tensoring with matrix algebras over $\boldsymbol{C}$. His reduction of the $n \times n$-case to the 2×2-case is the same as ours, but his proof of the 2×2-case (Lemma 2, p. 415) contains a definitely false inequality for the spectral radius and thus is wrong.

Let Γ be a locally compact group, H a closed normal subgroup and let $G = \Gamma/H$ be the quotient group. Assume that we have a measurable cross section from G into Γ. Then in [6] and [7] it was shown that $L^1(\Gamma)$ is isomorphic with a generalized L^1-algebra $L^1(G, L^1(H); T, P)$. Thus our result on groups will be a consequence of a more general one: Let G be a finite group, $\mathscr{A}$ a Banach *-algebra on which G acts and let P be a factor system of G with values in the unitary multipliers of $\mathscr{A}$ (see [6], [7]). Then the