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GENERATORS FOR EVOLUTION SYSTEMS WITH QUASI CONTINUOUS TRAJECTORIES

JAMES V. HEROD

With G a normed space, this paper provides conditions on a nonlinear function A from $R \times G$ to G in order to insure that if P is in G then there will be a (not necessarily continuous) solution Y for

$$Y(x) = P + \int_0^x d_t A(t, Y(t)) .$$

Early work in the study of the Stieltjes integral equation

$$M(x, z) = 1 + \int_x^z dFM(I, z)$$

was done by H. S. Wall [25] and T. H. Hildebrandt [8]. In Wall's paper, F is a continuous matrix valued function which is of bounded variation on each finite interval. Hildebrandt dropped the requirement of continuity and used a modified Stieltjes integral. J. S. Mac Nerney carefully analysed these ideas in a series of papers which led to the fundamental relationships found in [15], [16], and [17].

The papers [15] and [17] establish two classes OA and OM of functions and a one-to-one pairing of the classes made possible through a continuously continued sum, a continuously continued product, and a Stieltjes integral equation. In [17], if V is in OA, M is in OM, S is a linearly ordered set, and P is contained in a complete, normed, Abelian group, then V and M are related by $M(x, y)P = {}_x\prod^y [1+V]P$, $V(x, y)P = {}_x\sum^y [M-1]P$, and $M(x, y)P = P + \int_x^y VM(I, y)P$.

The results in [15] may be identified with analogous results in ordinary differential equations associated with nonautonomous, continuous, linear systems and [17] may be identified with Lipschitz systems. An indication of the nature of the generality obtained in the Stieltjes integral equation theory is found in [16], or in David L. Lovelady's discussion of interface problems [11, p. 184], or in a recent paper by Robert H. Martin [20] which investigates a linear operator equation and which identifies the linearly ordered set as the positive integers. Additional results related to [15] were found by B. W. Helton and Davis-Chatfield (see [2] or [3]). Also, this author determines a characterization of subsets of the two classes OA and OM which give rise to invertible evolution operators M in [4], for the linear case, and in [7] for the nonlinear (but Lipschitz) case.