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MAXIMAL QUOTIENT RINGS OF GROUP RINGS

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Let $F[G]$ be the group ring of a group G over a field F, and Δ the subgroup of G consisting of those elements with only finitely many conjugates. Let $Q(R)$ denote the maximal (Utumi) quotient ring of a ring R. This paper proves: (1) If H is a subnormal subgroup of G, $Q(F[H])$ is naturally embedded as a subring of $Q(F[G])$. (2) $Q(F[\Delta])$ contains the center of $Q(F[G])$. (3) If $F[G]$ is semiprime with center C, $Q(C)$ is the center of $Q(F[G])$. These results are analogues of theorems of M. Smith and D.S. Passman for the classical (Ore) quotient ring.

1. Introduction. Rings are associative and have a unit, and modules are unitary. Group rings will always be over fields, and we follow the definitions and notation of [5] for group rings and of [3] for quotient rings. In particular, if $F[G]$ is the group ring G over F, then

$$\Delta = \Delta(G) = \{g \in G: g \text{ has finitely many conjugates}\};$$
$$\Delta^+ = \Delta^+(G) = \text{torsion subgroup of } G;$$
$$\theta: F[G] \to F[\Delta] \text{ is the natural projection.}$$

If R is a ring, $Q = Q(R)$ is the maximal quotient ring of R.

There are many quotient rings which can be associated with a ring R. The two which have received the greatest attention are the classical (Ore) quotient ring and the maximal (Utumi) quotient ring. The classical quotient ring has a relatively straightforward description, but it is only defined for rings which satisfy the so-called Ore condition. In contrast the maximal quotient ring is less easy to describe but is defined for all rings. In both cases there are distinct notions of left and right quotient rings and we will always consider left quotient rings.

For group rings the classical quotient ring has been studied by Herstein and Small [2], Passman [5, 6], M. Smith [7], and P. F. Smith [8], and the maximal quotient ring has been studied by Burgess [1].

This paper investigates the relationship of the maximal quotient rings of group rings, subgroup rings, and the centers of group rings. The object is to obtain for the maximal quotient ring analogues of theorems of Passman and M. Smith on the classical quotient ring. Their techniques are used for the group ring arguments while the quotient ring arguments reflect the formalism of the maximal quotient ring.